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ABSTRACT

The critical need for greater numbers of trained technicians provided the general impetus for developing this approach to teaching the math skills which are needed for basic science and technology. This instructional system, which has been developed over a 4-year period, focuses on the average and below average students who enroll in industrial technology as opposed to engineering technology programs. It incorporates a learning center with separate treatment for fast, regular and slow learners, programed instruction, and teacher aides. Although it was developed primarily for teaching mathematics in a technical institute, the system has been used in other institute courses and in three other institutes, two high schools, and one junior high school. The system has implications for individualized instruction both for mathematics education and for education in general. (CH)

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MATHEMATICS FOR THE MAJORITY

A System of Instruction for Technical Mathematics

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ABSTRACT

This report summarizes the four-year development of a system of mathematics instruction for average and below-average learners. The system has been designed to communicate the math skills which are needed for basic science and technology. In this system, the content is communicated by programmed materials which incorporate what is known about the learning process. The use of programmed materials is supported by continual diagnostic assessment and personal tutoring. This novel system, which has been successful in communicating with both college and high school students, gives a new optimism about the learning ability and motivation of average and below-average students. Therefore, it has broad implications for both math education and education in general. For math education, it offers a new method of instruction and suggests a new minimum curriculum for all elementary and secondary school students. For education in general, it suggests that future attempts at instructional improvement should be concentrated on developing similar systems of instruction.

The major effort of the project has been devoted to the two-semester Technical Mathematics course for industrial technicians at the Milwaukee Area Technical College. After one year of experience with 70 pilot students, the project has been responsible for the instruction of all entering technical students (roughly 500 per year) during the past three years. During its four-year development, the system of instruction has gradually evolved into the use of a Learning Center which offers separate treatments for fast, regular, and slow learners. The operation of the Learning Center has become more efficient and economical because of the use of teacher aides and clerical personnel. Besides changing the content of the course radically to make it more relevant to the needs of industrial technicians, the dropout rate in the course has been substantially reduced and the achievement level of the students has been substantially increased. The reaction of the students to the system of instruction has been overwhelmingly positive. Though the teachers have been required to fill a new role, their reaction has become progressively more positive during the course of the project.

Though not specifically designed for high school students, the learning materials have been used on an experimental basis in various high schools in the Milwaukee area. Ordinarily the experimental courses have been offered as an alternate to General Mathematics courses. Data is presented from experimental classes at Pius XI High School and West Division High School. The results of comparisons between the experimental classes and college-preparatory classes on a basic algebra test are reported. The experimental classes compared very favorably. An assessment of the arithmetic skills of entering freshmen at Pius XI High School is also reported. The reaction of high school students and teachers to the system of instruction has been positive.

ABOUT THE AUTHORS

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CHAPTER I

INTRODUCTION

This report will summarize the history and current status of a project whose goal has been the development of a system of instruction for Technical Mathematics. In this chapter, we will discuss the origin of the project, summarize its history and goals, and identify some of the problems it faced.

Origin of the Project

The need for improved mathematics instruction for industrial technicians is based on a growing need for industrial technicians, plus the fact that technician training is plagued with a high dropout rate. After discussing the need for the project, a brief summary of its history and goals will be given.

Need for Technicians.

During the past thirty years, we have experienced the rapid growth of a technological society, and all indicators suggest that this growth will continue at an accelerated rate. A rapid change of this type is not without its pitfalls, one of which is the fact that the job market has changed radically. The need for skilled personnel has rapidly increased while the need for unskilled personnel has rapidly decreased. These changing demands of the job market would be no problem if training had kept pace. Unfortunately, it has not. Therefore, we are faced with the dual problem of a lack of skilled personnel to sustain the growth of a complex society, and more unskilled personnel than the job market can absorb.

One of the critical manpower needs in the United States is the need for trained technicians. Though the term "technician" is not well defined, technicians who complete a two-year, post-high school program can be broadly subdivided into two categories: engineering technicians and industrial technicians. Engineering technicians are trained to work closely with engineers and scientists engaged in research and development. They hold job titles like Engineering Assistant, Junior Engineer, Research Assistant, or Engineering Aide. Industrial technicians are trained to work more directly on the production aspects of industry. They hold job titles like Instrument Technician, Production Control Technician, Electronic Tester, Service Technician, Laboratory Technician, Quality Control Technician, Numerical Control Technician, and Detail Draftsman. On the continuum between engineers and craftsmen, engineering technicians are closer to engineers and industrial technicians are closer to craftsmen.

Of the two types of technicians, engineering technicians are clearly the more elite. This fact is reflected in their training which is much more academically oriented than that of the industrial technicians. Whereas the emphasis for engineering technicians is more on theory than on manipulative skills with instruments and devices, the emphasis for industrial technicians is just the opposite. Even though industrial technicians must learn some theory, they do not learn it with as much depth as the engineering technicians do. Despite the fact that engineering technicians are more elite, the majority of technicians in the United States are industrial technicians.

According to the U. S. Bureau of Labor Statistics, the need for trained technicians is critical. According to their projected figures, there will be a shortage of 350,000 engineering technicians by 1975 if the current ratio (0.7 to 1) of technicians to engineers and scientists is maintained. If this ratio is increased to 2 to 1 (as the American Society for Engineering Education recommends), there will be a shortage of 1,000,000 engineering technicians by 1975. Taking into account the fact that engineering technicians comprise only a minority of the total technician group, the comparable shortage of industrial technicians will be overwhelming. If this need is not met, it is difficult to see how our industry can continue to thrive and grow.

Technician Training - Milwaukee Area Technical College (MATC).

The Milwaukee Area Technical College (formerly the Milwaukee Institute of Technology) is one of the largest public technical schools in the United States. It has been training engineering technicians since 1924 and industrial technicians since 1952. The vast majority of students (a ratio of about 10 to 1) enroll in the industrial programs. For engineering technicians, programs are offered in electrical, industrial, mechanical, and tooling. For industrial technicians, all of the following programs are offered:

Air Conditioning and Refrigeration
Architectural
Chemical
Civil: Highway
Civil: Structural
Combustion Engine
Dental Laboratory
Electrical: Communications
Electrical: Computer
Electrical: Electronics
Electrical: Instrumentation
Fire Fighting
Fluid Power
Mechanical: Design
Mechanical: Manufacturing
Metallurgical
Photo Instrumentation
Photography
Printing and Publishing

Although 19 different programs are offered, 74% of the students in September, 1968 enrolled in one of the following areas: Electrical (29%), Civil (21%), Photography (13%), Mechanical (11%).

The fact that engineering technicians are a more elite group is obvious from a comparison of the prerequisites for the two programs. For the engineering technicians, the high-school prerequisites are three semesters of algebra, two semesters of geometry, one semester of trigonometry, and two semesters of either physics or chemistry. For the industrial technicians, the high-school prerequisites are two semesters of algebra and two semesters of geometry, with no science course required. (Even these prerequisites are occasionally waived.) Obviously, the engineering technician programs generally attract a more apt and better prepared student.

A comparison of the mathematics courses in each program substantiates the fact that engineering technicians are given a more theoretical training. In their first year, they take the equivalent of college algebra, trigonometry, and analytic geometry. In their second year, they take calculus and differential equations, and their more advanced technical courses assume an understanding of calculus. The mathematics courses for the industrial technicians are much less substantial. Three of the programs (Dental, Photography, Printing and Publishing) require only a one-semester course, which includes basic algebra, slide rule and calculations, graphing, logarithms, and an introduction to trigonometry. The remaining programs include the topics listed above, plus further topics in algebra, graphing, trigonometry, logarithms, and exponentials. Some of the electrical programs require a third semester of mathematics which is an introduction to calculus. Of all technical courses, only a few in the electrical technology programs assume an understanding of basic calculus.

A Dropout Problem With Industrial Technicians.

A major concern about the two-year training programs for the industrial technicians has been the high dropout rate. Of the 500 to 550 students who enroll each fall, roughly 35% actually complete the two-year program and obtain an associate degree. Part of this dropout rate is understandable. Since most high schools do not teach technical courses, many of the students who enroll have no idea what a technician is. Therefore, some simply change their minds after their initial contact with the technical courses. Others enter the Armed Forces or quit because of some job conflict. Furthermore, the college maintains an "open door" policy by accepting any student with the prerequisites in spite of his high-school rank. Obviously, some of the students who enroll simply do not have the ability to complete a two-year technical program. They either transfer to a more craft-oriented program or disappear. But in spite of these understandable reasons, the dropout rate still seems high, especially when the growing need for industrial technicians is taken into account.

Though not the only source of difficulty in the two-year training programs, the mathematics course was clearly identified as one of the major sources. Technical Mathematics is a two-semester core course for all students in the industrial-technician curricula. Four credit hours are given for each semester. It is a critical course in the curricula since the success of the mathematics instruction affects the success of the instruction in most technical courses. The results in the Technical Mathematics course at MATC were typically unsatisfactory. Aware of the nature and magnitude of the problems encountered by the students in this course, in 1960 the college selected it to be the first locally produced credit course to present over its educational television station. In spite of some improvement, the results were still not good. Coupled with a 40% dropout in the first semester and a 20% dropout of the remaining students in the second semester, the achievement level of students on final exams rarely exceeded a mean score of 55% or 60%. Though grades were curved so that only a small percentage of the remaining students failed, a passing grade did not mean that a student possessed the fundamental skills required for his technical courses. Because of the high dropout rate and low level of achievement, this course continued to be the source of many complaints from both students and technical teachers.

History and Goals of the Project.

With the hopes of developing a system of instruction which might improve the success of the Technical Mathematics course, a grant proposal was written and submitted to the Carnegie Corporation in December, 1964. A $200,000 grant was approved in December, 1964; the project began in June, 1965, and it is still in progress. (The Carnegie funds were exhausted in February, 1968. Since that time, the project has been jointly supported by MATC and the Wisconsin State Board of Technical, Vocational, and Adult Education.) Since the project began, three pilot classes (about 75 students) were taught in the fall semester of 1965-66, and all technical students (about 500 per year) were taught during both semesters in 1966-67, 1967-68, and 1968-69.

Though the primary effort has been concentrated on the development of a system of instruction for the Technical Mathematics course, the materials have been used in other courses within MATC, in three other technical colleges in Wisconsin, in two local high schools, and in one junior high school. (Approximately 1,500 students used all or parts of the materials during the 1968-69 school year.) There are two major sections in this report. The first describes the system of instruction developed for the Technical Mathematics course at MATC and discusses the results obtained. The second discusses the results obtained with experimental classes at two Milwaukee high schools, Pius XI and West Division. Some brief comments will also be made about other uses of the materials.

In developing a system of instruction for Technical Mathematics, the project staff has had three goals:

(1) to develop a course content which is more relevant for technician training,

(2) to increase the level of achievement, and

(3) to reduce the dropout rate.

Underlying the whole effort has been the hope that attaining the three objectives above in the math course would reduce the dropout rate and upgrade the achievement level in the technology curricula.

An Analysis of Conventional Technical Mathematics Instruction

The problems encountered in the Technical Mathematics course at MATC are not unique to our institution. Instructors from other technical institutions in Wisconsin and elsewhere generally report similar, if not worse, results. In this section, we will analyze a conventional Technical Mathematics course to identify the various problems which are involved in it. Though this analysis will be based on past experience at MATC, it will reveal problems which are encountered in Technical Mathematics courses anywhere.

To analyze a conventional Technical Mathematics course, we will discuss the following components: (1) Core-course problems, (2) Students, (3) Mathematics teachers, (4) Expectations of technology teachers, (5) Content and textbooks, and (6) Lecture-discussion method. Though not necessarily mutually exclusive, each component will be discussed separately. Since this analysis will identify many of the deficiencies in a conventional Technical Mathematics course, it can serve as a standard of comparison for the new instructional system.

Core-Course Problems.

As a core-course, Technical Mathematics must serve students from all technical majors. The mathematical needs of the various majors are quite diverse, both in terms of topics and the level of sophistication required in topics. Therefore, it is difficult to decide on a core content which fills the needs of the students in all technologies. On the other hand, it would be almost impossible to design a unique course for students in each technology. The content has to be somewhat of a compromise.

Besides the content problem, there is a sequencing problem. Though designed to prepare students for their technical courses and a science course, the math course is taught simultaneously with many of these courses. Given the restrictions of the necessary sequencing of mathematical topics and the speed with which the students can learn, it is almost impossible to treat all topics before they are encountered in the technical courses. Topics which are taught "too late" are a constant source of complaint from the students and technical teachers.

Students.

Though the math skills which the students have when the course begins are extremely heterogeneous, for the most part their math skills are seriously limited. (See "Characteristics of the Students," pp. 10-13.) The entrance requirement of one year each of high-school algebra and geometry is no guarantee that the students either learned or remember the fundamental skills taught in those courses. Not only do many of them have serious deficiencies in topics which a Technical Mathematics course would like to assume, but many of them are also slow learners. Since the entry skills of the students have decreased slightly during the four-year history of our project, there is apparently nothing new happening in the secondary schools to raise anyone's hopes about better-prepared students in the immediate future.

Besides the entry-skills problem, there is also a problem with the attitude of many of the students. Despite the fact that they have received a passing grade in two years of college-preparatory math, many of the students have had the equivalent of a "failure" experience with mathematics, and they are astute enough to recognize this fact. Therefore they enter the Technical Mathematics course with an anxious, defeatist attitude.

Mathematics Teachers.

The teachers assigned to the Technical Mathematics course are usually junior-college mathematics teachers. As junior-college mathematics teachers, they are much more familiar with the mathematical needs of a student who is preparing to take further math courses than they are with the mathematical needs of an industrial technician. Many of them, in fact, have had little training in either science or technology. And since they frequently look upon an assignment to the Technical Mathematics course as a necessary evil, the probability of their spending much time to examine the mathematical needs of technicians is very low. As a result, they have no basis for distinguishing between relevant and irrelevant topics.

The attitude of the teachers towards the course and the students in it is frequently negative. There are various reasons for this negative attitude. Many teachers are dissatisfied with the low level of content which they are forced to teach. And added to this unsatisfactory level of content, they are faced with many students who begin the course with deficiencies in assumed topics, low ability-levels, and an apparent lack of motivation. Though recognizing the fact that many of the students need extensive remedial work, they have no mechanism by which this remedial work can be accomplished. Furthermore, the lecture-discussion method of instruction which they use is no more successful than it has ever been with lower-ability students. Frustrated by their lack of success, their usual defense is to blame the students by calling them "stupid" or "unmotivated." This hostile, negative attitude is increased by the fact that there are many complaints about the course from students and technology teachers.

Expectations of Technology Teachers.

Technology teachers can be roughly divided into two distinct groups, skilled tradesmen and engineers. In terms of their expectations about mathematical skills for their students, teachers in these two groups differ considerably because their emphasis on theory differs. Whereas the skilled tradesmen frequently do not even use the full range of mathematical skills which the students have, the engineers frequently want a range of skills which is beyond the capacity of many of the students, at least with the time allotted for mathematics instruction. The engineers frequently overestimate the learning ability of the students because they fail to recognize the vast difference in learning ability between technicians and engineers. Many of the engineers are not fully aware of the low level of entry skills in the mathematics course. This is especially true of those who teach only second-year courses, since many of the slower learners have dropped out by that time. Therefore, their expectations for the mathematics course are unrealistic.

Besides unrealistic expectations about the content which can be reasonably covered in the mathematics course, many technology teachers are also unrealistic about the level of learning and level of retention of mathematical skills. When a mathematics topic is needed in a technical course, the technology teachers frequently do not review the topic. By not reviewing, they are implicitly assuming a mastery level of achievement plus a high level of recall. This assumption is somewhat unrealistic about any course, even their own. It is especially unrealistic for slower learners whose retention rate is far from perfect.

Content and Textbooks.

Since the training of industrial technicians on a large scale is a relatively recent phenomenon, Technical Mathematics courses have not been in existence for a long time. A Technical Mathematics course is somewhat out of the mainstream of mathematics education, since its goal is to teach those skills which are required for elementary science and technology. It is a terminal course and not a course whose goal is preparation for further mathematics courses. Therefore, it must be selective in its content because there are clearly enough relevant topics to warrant the exclusion of irrelevant ones. It must emphasize topics such as fluency with numbers, slide-rule calculation, and an ability to handle formulas and derivations, topics which unfortunately are not emphasized in traditional mathematics courses.

Since Technical Mathematics teachers are frequently uncertain about the proper content for the course, they must rely on the available textbooks for this guidance. However, the content in the available textbooks is far from satisfactory. The authors seem to be either mathematics teachers or technical teachers who are geared more towards the training of engineering technicians. Therefore, the textbooks are geared for a type of technician who has high entry skills in mathematics and who needs a more substantial mathematical training. Not only do the textbooks assume a mathematical background which the entering industrial technician student usually does not have, but they reflect the fact that the authors themselves were unable to make a clean break from traditional mathematics education. That is, the textbooks usually look like watered-down versions of college courses because they not only include too many irrelevant topics and unnecessary complexities in relevant ones but they exclude or underemphasize many relevant topics.

Forced to use textbooks of this type since no others are available, the teacher is faced with many dilemmas. Recognizing the fact that the students have serious deficiencies in topics which the textbook assumes, he must decide whether he has the time, energy, and inclination to attempt the remedial work. Even if he wants to attempt it, suitable materials are usually not available, and the students might neither want nor be able to do remedial work concurrently with the work in their regular class. During the course itself, if the teacher suspects that a topic or certain complexities of a topic are irrelevant, the textbook may not be written in such a way that the irrelevant parts can be skipped. And if he suspects that a relevant topic is either underemphasized or ignored, he may not have the time or ability to write suitable supplementary materials. In other words, he is a virtual slave to the textbook unless he is willing to exert an extraordinary amount of additional effort. Usually he does not have enough time for an extraordinary effort of this type.

Lecture-Discussion Method.

In the typical Technical Mathematics course the lecture-discussion method is used in conjunction with a conventional textbook. Though it is becoming more and more obvious that this method is not very successful in communicating mathematics, science, or technology to average and below-average students, it is still used for various reasons. For one thing, many teachers are loath to try any other method, not only because all of their experience and training is with the lecture-discussion method but because it is the method which they enjoy using. For another thing, even those who would like to try a different method find that other methods are not available, and they themselves do not have the time and energy to develop one.

The major criticism of the lecture-discussion method for average and below-average students is the fact that it is a dehumanizing experience for many of them. It is dehumanizing because the students are frequently unsuccessful in their attempt to learn, and the method itself is not geared to remedy this lack of success. With average and below-average learners, the lecture-discussion method has many identifiable deficiencies. In the following paragraphs, we will discuss some of the more obvious ones, such as lack of communication, lack of individual attention, too heavy an emphasis on teacher activity, inadequate assessment, "curving" grades, and teacher attitudes.

Education is a matter of communication, and communication with students presupposes a knowledge of their learning processes. Unfortunately, the authors of textbooks and teachers usually do not have a sophisticated knowledge of the learning process since little time is devoted to studying the learning process in teacher-training. This lack of knowledge of the learning process is frequently masked when teaching high-ability students, since most of them are able to compensate for faulty communication. However, it becomes immediately apparent with average and below-average students. Since the latter type of student cannot compensate for a lack of attention to details, for definitions which are too abstract, and for an assumed degree of transfer which does not occur, his learning process immediately breaks down.

The lecture-discussion method places too heavy an emphasis on teacher activity and too little an emphasis on student activity. As most teachers would readily admit, textbooks are not written in such a way that the students could learn exclusively from them. Therefore, the instruction becomes highly correlated with the teacher's activity. Over and above the fact that lectures are not always clear, other problems occur. Students do not always pay attention, many times because the lectures are boring. Students are also absent. When a student is either absent or not paying attention, he really misses a segment of the instruction, and ordinarily there is no way for him to obtain this missed instruction since the method does not provide an opportunity for it.

The lecture-discussion method is a group method of instruction; it has never pretended to be a method which offers the possibility of coming to grips with the learning process of individual students. When using this method, the teacher spends most of the class time lecturing or discussing problems with the students as a group. He does not get enough feedback from each individual student to have a running assessment of how well the individuals are attaining the learning objectives. Tests cannot be given too frequently or the progress of the class bogs down. And tests frequently have the philosophy of "grading" rather than the philosophy of "diagnosis." That is, the items are frequently designed to discriminate among students rather than to assess the attainment of learning objectives. Even when the items do assess the learning objectives and consequently provide information about the progress of each student, the teacher has little opportunity to do anything for the students who are not making adequate progress. Not much class time can be devoted to remedial work since other topics must be covered, and the amount of time outside of class which can be devoted to tutoring is limited. Unfortunately, in a sequential subject like mathematics, unremedied breakdowns in the learning process at any point are a prelude to future disaster.

The lack of success of the lecture-discussion method with average and below-average students is hidden by the fact that course grades are curved. For example, in the fall semester of 1965, the following results were obtained in the Technical Mathematics classes in which this method was used. After a 39% student dropout during the semester, the mean score on an 85-item final exam for 295 students was 57%. After a statistical analysis, the achievement testing department suggested that a student with a score of 18% (15 correct out of 85) be assigned a "D" for that test. Even at that, 22 of the 295 students scored below 18% and were given an "F". Since the items in the test assessed basic learning objectives, it should be clear that a "passing" grade on that test had little relationship to the attainment of learning objectives for many students. This type of factual information, which is a fairly objective and honest assessment of the success of the instruction, is lost sight of when the scores are "curved" into the five traditional "letter" categories.

The lecture-discussion method is geared more towards "covering content" than it is towards "the attainment of learning objectives." Though course objectives are frequently unrealistic, teachers can easily become compulsive in plowing through the content even though they realize that many students are not learning. Even teachers who are seriously concerned about learning soon find that the method is working against them. There is no mechanism for handling remedial work, lack of attention, or absentees. Since the method is not designed to give the teacher an opportunity to control the learning process of each individual student, even dedicated teachers have to abandon that goal. Therefore, many teachers adopt a relatively passive attitude about student learning. They enjoy the learning which occurs and ignore that which does not occur.

It is easy to see why the lecture-discussion method is a dehumanizing experience for many average and below-average students. Unable to compensate for a type of instruction which frequently does not communicate with them, no other method of instruction is provided. Needing constant individual monitoring of their learning processes, they are caught in a system in which the possibility for personal interactions between teacher and student is severely limited. As a result, they are frequently confused throughout much of a course, and even though they are happy to get the passing grade which "curving" provides, they are smart enough to realize that this passing grade does not reflect an adequate level of learning. Since honest appraisal and criticism of the system are not common, students are forced into a position of self-blame. Most courses become one more piece of evidence which confirms their belief that they are dull and cannot learn. Though a student's self-blame undermines his self-confidence and sense of personal worth, teachers and administrators do little to prevent it because the only other alternative is to accept some of the blame themselves. Unfortunately, the teaching profession has not yet shown much willingness to accept responsibility for the outcome of its efforts.

Characteristics of the Students

Though brief, this section is very important. Information about the students is very important because of the philosophy of education which has been adopted by the project staff. There are two distinct philosophies which can underly a mathematics course (or any course for that matter). The first philosophy permeates many mathematics courses at both the high school and college level. This philosophy views each course as a necessary proving ground and preparation for further courses. The content and pace of the instruction are determined by what is expected in the next course in the sequence. Aside from some prerequisite courses, which frequently are no guarantee of prerequisite skills, little consideration is given to the entry skills of the students or the speed with which they can learn. Students either succeed or not. Those who prove themselves can go on; those who do not prove themselves cannot go on. The responsibility of proving themselves is placed directly on the students. The second philosophy views a mathematics course as a process of taking the students from where they are to where they can reasonably get within the time limits of the course. If this second philosophy is adopted, much more attention must be given to the characteristics of the entering students. For example, something must be known about their entry skills, their previous academic success, their work habits, and the speed at which they can learn.

The second philosophy was adopted by the project staff. The major reason, of course, had to do with the goals of the project. To reduce the dropout rate and raise the level of achievement in the Technical Mathematics course, it seemed clear that the entry skills and learning ability of the students had to be taken into account. This type of background information is especially needed in a program like technician training at MATC, which has an "open-door" policy aside from the following two minimal prerequisites: (1) that each student must have earned a high-school diploma, (2) that each student must have earned a passing grade in one year of algebra and one year of geometry.

In this section, we will discuss the general characteristics of the entering students and report their entry skills in terms of two mathematics pre-tests. Though the information was obtained from the students in the 1968-69 school year, it is typical of the students who have enrolled during the four-year history of the project. In fact, discussions with teachers from other technical-training institutions in Wisconsin suggest that it also typifies their entering students.

General Characteristics.

The general characteristics of the 480 students who enrolled in Technical Mathematics in September, 1968 are summarized in Appendix A. Some of the more salient points are:

(1) 72% were either 17, 18, or 19 years of age.

(2) 80% reported that they had not previously attended some type of college.

(3) 72% ranked in the bottom half of their high school classes with a median rank at the 32nd percentile.

(4) In the two years of required high-school math, the percent receiving a "D", "U", or "not taking" increased from 38% in the first semester of algebra to 49% in the second semester of geometry.

(5) 54% did not take more than the two required years of college-preparatory math.

(6) Only 34% claimed some ability to use a slide rule; only 34% took a high school physics course.

(7) Except for English, their ACT scores compared favorably with the norms for junior colleges.

The entering students are clearly very heterogeneous in terms of their ability and their level of high school achievement. Though their ability level, as measured by ACT scores, compares favorably with the ability level of students entering junior colleges, in general they have not had a history of academic success. This lack of success can be attributed either to their own lack of motivation, to the school system's inability to communicate with them, or to a combination of the two. Whatever the reason, many have not been strong academic students, and it is safe to assume that many have not developed the type of work habits which are needed for success in academic courses.

Pre-Tests in Arithmetic and Basic Algebra.

At the beginning of the fall semester (September, 1968), all students in the Technical Mathematics course were given two pre-tests, one in arithmetic and one in algebra. Each test used the constructed-response format. Each test was designed for administration in one 50-minute period. The results of these tests are summarized on the next page.

Pre-Test in Arithmetic. A copy of the arithmetic pre-test is given in Appendix B-1. This 50-item test covered the following general topics: whole numbers, decimals, percents, number system, number sense, and fractions. The overall mean and median for 475 students were 64% and 66%, respectively. A distribution of scores and an item analysis are given in Appendix B-2.

In the table below, the mean percent "correct," "incorrect," or "not attempted" for each sub-section is given:

STUDENT PERFORMANCE ON SUB-SECTIONS OF ARITHMETIC PRE-TEST
MATC TECHNICAL MATHEMATICS - SEPTEMBER, 1968

Sub-Sections		Correct	Incorrect	Not Attempted
Whole Numbers	(4 items)	84%	16%	0%
Decimals	(4 items)	76%	23%	1%
Percents	(6 items)	72%	22%	6%
Number System	(10 items)	59%	38%	3%
Number Sense	(4 items)	75%	24%	1%
Fractions	(22 items)	56%	31%	13%

The instructions for items 43, 44, and 45 in the "fractions" section were not clear to the students. This lack of clarity is reflected in the percent of students (28%, 31%, and 47%, respectively) who did not attempt these items, as shown in Appendix B-2.

Pre-Test in Algebra. A copy of the algebra pre-test is given in Appendix C-1. This 30-item test covered the following general topics: operations with signed numbers, powers of 10, addition of algebraic fractions, non-fractional equations, fractional equations, and formula rearrangement. The overall mean and median for 471 students were 37% and 30%, respectively. A distribution of scores and an item analysis are given in Appendix C-2.

In the table below, the mean percent "correct," "incorrect," or "not attempted" for each sub-section is given:

STUDENT PERFORMANCE ON SUB-SECTIONS OF ALGEBRA PRE-TEST
MATC TECHNICAL MATHEMATICS - SEPTEMBER, 1968

Sub-Sections		Correct	Incorrect	Not Attempted
Signed Numbers	(6 items)	52%	45%	3%
Powers of Ten	(2 items)	20%	63%	17%
Algebraic Fractions	(2 items)	33%	61%	6%
Non-Fractional Equations	(6 items)	60%	26%	14%
Fractional Equations	(6 items)	19%	35%	46%
Formula Rearrangement	(8 items)	28%	36%	36%

17 of the 30 items were identical to items given in a similar pre-test in September, 1967. The percent "correct" on these 17 items in 1967 is reported in Appendix C-2 along with the item analysis for 1968-69. The average percent "correct" on the 17 items was 3.5% lower in 1968-69 than it was in 1967. There were gains on only 2 items.

The items in both pre-tests check basic skills which could reasonably be assumed to be included in high-school mathematics courses. Therefore, the two median scores (66% in arithmetic and 30% in algebra) were disturbing. The students were especially weak in fractions, fractional equations, and formula rearrangement, topics which are highly relevant for basic science or technology. The high percentage of students who did not attempt the "fractional equations" and "formula rearrangement" items suggests that these topics are not an integral part of many high-school mathematics programs. The low scores on the algebra pre-test could be interpreted in either of two ways: (1) that a tremendous amount of forgetting had occurred, or (2) that the students never really learned these basic skills. The project staff chose the latter interpretation. This choice has been corroborated by the results on similar tests given to students in Milwaukee high schools. (See Chapter 4.) The low scores do not speak well for the quality of high-school mathematics instruction, at least for the type of student who enrolls in technician training programs.

CHAPTER 2

SYSTEM OF INSTRUCTION

The system of instruction which has been developed is a radical departure from the conventional lecture-discussion method. Though this system has gradually evolved and continues to do so, its major goal has been to remedy some of the obvious deficiencies of the lecture-discussion method for average and below-average learners. The system has been designed to handle a large number of students (about 500) in the ongoing operation of a large, complex institution. An effort has been made to incorporate in it the following features: individual attention for the students, relevant topics, learning materials which are based on the known principles of learning, careful assessment, and a type of self-criticism which offers the possibility of constant improvement.

Aside from the Project Director, who is an experimental psychologist, other members of the project staff have been members of the mathematics department at MATC. At the beginning of the project, each staff member participated in writing instructional materials and tests and in handling classes of students. During the course of the project, however, there has been a general trend towards specialization. At the present time, one staff member is responsible for writing the learning materials, a second is responsible for content and tests, and a third is responsible for the organization and implementation of the teaching system. The other staff members are mainly responsible for handling students in the various classes. Since the staff member who is responsible for the organization and implementation of the teaching system is also responsible for a full load of classes, there are only two non-teaching members of the staff. A small staff has advantages and disadvantages. A major advantage is that decision-making is not paralyzed by the discussion of many diverse ideas. A major disadvantage is that the production of instructional materials proceeds rather slowly.

In order to describe the system of instruction adequately, it will be discussed in terms of the following major components:

I. Course Content
II. Learning Materials and the Use of Learning Principles
III. Assessment Instruments
IV. Classroom Procedure

In this chapter, one section will be devoted to each of these four major components.

I. COURSE CONTENT

The purpose of a Technical Mathematics course is to teach the mathematics skills which a technician actually needs for his training and his future job. Unlike many mathematics courses, it is a terminal course. Though not precluding the possibility that the student will take more mathematics, it should not be specifically designed for that purpose. Therefore, a philosophy like "the more math the better" is completely irrelevant when making decisions about possible topics. Only topics which are relevant for technicians should be included. The principle of "relevance" must be maintained for three reasons: (1) there are many relevant topics, (2) the average student is not a fast learner, and (3) the average student does not have a solid foundation in mathematics.

Ordinarily teachers rely on textbooks to determine the content of a course. As was mentioned earlier, the choice of topics in the available textbooks for Technical Mathematics is far from satisfactory. The textbooks assume skills which the entering technical student does not have. They include topics or complexities of topics which are irrelevant because they are only needed if the student is being prepared for further mathematics courses. They include more content than the students can learn in the time allotted for mathematics instruction in technical training. Therefore, a decision was made to design a completely new course with these criteria: (1) each topic must be relevant for technicians, (2) the content must begin at a level which coincides with the entry skills of the students, and (3) the instruction must proceed at a pace which coincides with the learning speed of the students.

In this section, we will discuss the procedure used to determine the course content. We will also discuss the content which was taught during the 1968-69 school year.

Information Obtained from Surveys.

There are two sources of information about the relevance of math topics for an industrial technician: the math which he actually needs on the job and the math which he needs to learn the theory in his technical courses. At the beginning of the project, separate surveys of industry and the technology teachers were made to tap each source of information. No survey of either type can be followed blindly when determining the content of a course. Therefore, after briefly discussing each survey, the project staff's philosophy for determining the content will be explained.

Survey of Industry. During the summer of 1965, a copy of the old course objectives for each semester was sent to various companies in the Milwaukee area. The companies selected were those who hire a large number of the technical graduates. The companies were asked to rate each objective as either "important," "somewhat important," or "unimportant." The rating was typically done by either personnel directors or engineers in some managerial position. Since the vast majority of the responses reflected a "the more math the better" philosophy, the survey obviously revealed the hope of employers rather than their estimate of the on-the-job math skills which their technicians need. Therefore, the survey was relatively useless.

Two more sophisticated surveys of the on-the-job math skills of technicians are available. They are: (1) The Role of Mathematics in Electrical-Electronic Technology, conducted by Melvin L. Barlow and William J. Schill in California in 1962, and (2) Mathematical Expectations of Technicians in Michigan Industries, conducted by Norman G. Laws in 1966. The project staff was aware of the first survey when the project began, the second was not available until a year after the project began. The California survey is exclusively devoted to electrical technicians, whereas the Michigan survey gives little emphasis to electrical technicians since they are not prominent in that area.

Surveys of Technology Teachers. The technology teachers at MATC were formally surveyed twice. The first survey was a written one identical to that sent to local industries. That is, each teacher rated each topic in the old course objectives as either "important," "somewhat important," or "unimportant." This first survey was conducted during the fall semester of 1965. It revealed that topics like higher-degree equations or the multiplication and division of polynomials are "unimportant" for all technologies. It also revealed that an introduction to calculus and topics which are required only for calculus (like complicated trigonometric identities or a formal treatment of the conic sections) are irrelevant for all technologies except for some programs in electrical technology. After the one-semester tryout with the pilot classes in the fall semester of 1965, a second survey of the technology teachers was conducted. In this second survey, each technology teacher was personally interviewed by a member of the project staff. The purpose of the interview was to reexamine the relevance of topics and investigate the inclusion of new topics. During the three-year period since these formal surveys were made, key technology teachers have been periodically interviewed on an informal basis.

Use of the Surveys. There are dangers in using either type of survey at its face value. A survey of on-the-job mathematical skills overlooks the mathematics required to understand the technology courses. Ordinarily, the mathematics required to learn the technology courses is much broader than that required on the job. A survey of technology teachers overlooks the fact that these teachers might be teaching their courses at the wrong level since there is no specific teacher-training for technology courses. Teachers with backgrounds in skilled trades tend to use less mathematics in their instruction; teachers with backgrounds in engineering tend to use more mathematics. The latter, in fact, sometimes teach at a level which is well beyond the capabilities of the students. Furthermore, most technology teachers have never taught mathematics, and they are unaware of both the necessary sequencing of mathematical topics and their relative difficulty.

Since more mathematical skills are needed to understand technology courses than are actually used on the job by technicians, the project staff relied heavily on the surveys of the technology teachers when determining the content for the course. The information from the technology teachers was tempered by the judgment of three members of the project staff who had a combined total of 30 years of experience with conventional Technical Mathematics instruction. These staff members took into account what they knew about the entry skills and the learning ability of the average student. They also tried to estimate the real need for each mathematical topic if all technology courses were taught at an appropriate level.

Learning Objectives, Learning Sets, and a Task Analysis.

Mathematics is a hierarchically organized body of knowledge from two different points of view. From one point of view of the mathematician, it is a hierarchically organized axiomatic-deductive system. From the point of view of the mathematics instructor, it is a hierarchically organized system of learning objectives which a student must master for continued success. Though these two systems do not necessarily conflict, they do not necessarily coincide either. That is, the structure which is best for the mathematician need not be the structure which is best for the original learning of a student. When designing mathematics instruction, attention must be focused on the student rather than the axiomatic-deductive system of the mathematician. A hierarchy of learning objectives or behavioral objectives for the student must be determined. There is no one unique hierarchy of major learning objectives. For example, either systems of equations or the basic trigonometric ratios can be learned first. Or when learning operations with signed numbers, either multiplication or subtraction can be learned first. However, some reasonable hierarchy of learning objectives must be followed.

When defining learning objectives, there is a distinct advantage to translating them into behavioral terms. By doing so, the objectives become specific and measurable. Though the project staff has always conceptualized the objectives in behavioral terms, they have not been written down in this form. In practice, the staff found that it was difficult to identify all subsidiary objectives before the actual writing of the materials. And after the materials were written, efforts to write the objectives in behavioral terms seemed to be a waste of time. Therefore, though the behavioral objectives are incorporated in the learning materials and reflected in the test items, the content will be described by means of traditional topic names.

After determining a hierarchy of learning objectives, the content was analyzed to identify the learning sets which the students must master in order to achieve each learning objective. A learning objective states what a student should be able to do; a learning set states what he must know in order to be able to do it. A "learning set" can be defined as a "verbalizable rule which is applicable to a class of stimuli." Here is an example of a learning set for some basic equations.

The verbalizable rule is: to solve for "x" in each equation, use the multiplication axiom, multiplying both sides by the reciprocal of the coefficient of "x".

The class of stimuli is:

$$\left.\begin{array}{l}2x = 7\\3x = y\\ax = b\end{array}\right\}\quad \text{or any similar equation.}$$

When a student knows this learning set, he is able to apply this specific rule to any stimulus in the class.

Since a learning set is a verbalizable rule, it can be communicated by means of verbal language and appropriate examples. Learning sets can range in complexity from "knowing that a mixed number (like $3\frac{1}{2}$) is an arbitrary shorthand for the addition of a whole number and a fraction (like $3 + \frac{1}{2}$)" to "knowing the strategy of solving for the variable 'c' in the equation $\frac{1}{a} = \frac{1}{b} - \frac{1}{c}$." More complex learning sets like the latter are based on the mastery of many subordinate learning sets. It should be obvious that there are thousands of learning sets in basic mathematics instruction.

Learning sets are identified by means of a task analysis. A task analysis is a detailed examination of all of the mathematical problems which are included in the learning objectives. The beneficial effect of such an analysis before any instruction is attempted cannot be overemphasized. Two such effects are:

(1) *Frequently, an analysis of the full range of stimuli to which a learning set is meant to apply determines the form of the verbalizable rule which is most general and transferable.*

For example, given a system of traditional equations and a system of formulas like the following:

$$\boxed{\begin{aligned} x + y &= 10 \\ 2x - y &= 8 \end{aligned}} \qquad\qquad \boxed{\begin{aligned} E &= IR \\ P &= EI \end{aligned}}$$

to solve the system of equations or to eliminate the variable "I" from the system of formulas, the first step in each case is the elimination of one variable. The "addition-subtraction" method is a quick method for eliminating "y" in the system at the left. However, the "addition-subtraction" method is not general because it cannot be used to eliminate "I" on the right. If the learning set is meant to be a general method which works for both sets of stimuli, a different method must be taught. Then some decision must be made about the need for teaching the "addition-subtraction" method. This decision will be affected by the actual usefulness of that method in non-contrived situations and by the amount of available instruction time.

(2) *A task analysis frequently identifies subordinate learning sets which might otherwise be overlooked.*

For example, in the process of solving for "b" in the equation

$$a - b = c$$

the following equation may be encountered:

$$-b = c - a$$

If a student cannot handle the latter equation because he has never been introduced to the learning set for doing so, the process of solving for "b" breaks down at that point. The lack of a learning set for the latter equation leaves a gap in the instruction which that particular student cannot bridge.

A detailed task analysis of this type is necessary in order to determine the complete hierarchy of learning sets which represent the course content. It frequently prevents unnecessary gaps in the instruction which occur because subordinate learning sets are overlooked. This identification of a complete hierarchy of learning sets is only a first step, however. The problem of communicating the individual learning sets to the students still remains. This latter problem involves a knowledge of the learning process and close attention to empirical results.

Content for 1968-69.

The project has been responsible for the math instruction of all technical students during both semesters for the past three years. Though the content of the course has changed slightly during this period, for the most part it has remained relatively stable. Admittedly, there are more topics which should be included, as will be discussed later. However, the staff has been more interested in student learning than in merely covering as many topics as possible. Therefore, the number of topics covered has been limited by the entry skills of the students, their speed in learning, their retention rate, and the time limits of the course. A brief outline of the topics covered during the 1968-69 school year is given below. A more detailed description of the same content is given in Appendix D.

Algebra

(1) Number line and signed numbers
(2) Non-fractional equations and formulas
(3) Numerical and literal fractions
(4) Fractional equations and formulas
(5) Systems of equations and formulas
(6) Radicals and radical equations and formulas
(7) Quadratic equations and formulas

Calculations and Slide Rule Operations

(1) Number system and number sense
(2) Powers of ten
(3) Rounding
(4) Estimation techniques
(5) Slide rule operations
  (a) multiplication
  (b) division
  (c) combined multiplication and division
  (d) squaring and cubing
  (e) square roots and cube roots
  (f) sine, cosine, and tangent
  (g) logarithms

Graphing

(1) Reading and constructing the following types of graphs:
    (a) linear and non-linear equations and formulas
    (b) exponential equations and formulas
    (c) sine waves
(2) Reading semi-log and log-log graphs
(3) Concepts of slope and intercepts
(4) Slope-intercept form of the straight line

Geometry and Trigonometry

(1) Basic geometric facts
(2) Areas and volumes
(3) Solving right and oblique triangles
(4) General angles
(5) Vectors and complex numbers
(6) Sine waves
(7) Basic trigonometric identities
(8) Inverse trigonometric notation
(9) Applied problems involving circles and half-tangents
(10) Degree and radian systems of angle measurement
(11) Angular and circular velocity

Logarithms and Exponentials

(1) Common and natural logarithms
(2) Finding powers and roots
(3) Laws of logarithms
(4) Evaluating logarithmic and exponential formulas
(5) Rearrangement of logarithmic and exponential formulas

General Features of the Content.

There are some general features about the course content which are worth noting. These general features, which are related both to the goals of the course and to the entry skills and learning ability of the students, will be discussed in this section.

Remedial Topics. Other than the basic arithmetic operations with whole numbers and decimals, no other topics were assumed. The algebra sequence, for example, starts with signed numbers, and it includes a lengthy review of fractions. The decision to minimize the number of assumed skills was based on the low entry level of the students. Therefore, many topics which otherwise would have required extensive remedial work for a majority of the students were included as an integral part of the content.

Use of "Modern Math" Principles. The principles of "modern math" are included in the content of the course. However, these principles are used with a different emphasis than they receive in a pure modern math course since the goals of a Technical Mathematics course are different than the goals of a pure modern math course. Whereas a pure modern math course emphasizes structure and proof, knowing the structure of mathematics and understanding proofs is an unreasonable goal for technicians. A technician needs the ability to use mathematics in elementary science and technology.

Therefore, he needs a more applied course with emphasis on manipulative skills and solving problems. An understanding of the basic principles of modern math is included in the content because the project staff felt that this understanding of basic principles is needed both to learn mathematics and to apply it properly. But even though the definitions, axioms, and principles of modern math are included, they are included solely as a means of developing manipulative skills. When the definitions, axioms and principles are introduced, they are intuitively justified by means of numbers. No attempt is made to give a formal insight into their general structure. Formal deductive proofs are also generally avoided or at least deemphasized. Therefore, the content is somewhat of a compromise between a pure modern math course with its emphasis on structure and proof and a purely applied math course which teaches rote-mechanical procedures.

Excluded Topics. Many topics which are an integral part of most math courses were excluded for the simple reason that they are not used in elementary science or technology. Their irrelevance was revealed by the surveys of the technology teachers. Some of the excluded topics are higher-degree equations, the multiplication and division of polynomials, a formal treatment of the conic sections, and complicated trigonometric identities.

Freedom from Closure. A sense of freedom from closure was also maintained with the topics which are included in the content. By "closure" we mean "including all aspects of any topic which is introduced." What we mean by "freedom from closure" can be best described by some examples. Here is a representative list of instances in which we have maintained this freedom:

(1) When the multiplication axiom is introduced and used, no statement is made that "0" should be excluded as a multiplier.

(2) When solving quadratic equations, methods involving completing the square and factoring trinomials are not taught.

(3) Operations with radicals are confined exclusively to square root radicals.

(4) In operations involving "j" ($\sqrt{-1}$), powers of "j" higher than j^2 are omitted.

(5) When solving radical equations, equations with extraneous roots and a discussion of extraneous roots are omitted.

(6) Though parabolas and hyperbolas are graphed, their formal geometric properties are not discussed.

(7) Though the solution of oblique triangles is taught, the law of tangents and half-angle formulas are not introduced, and the ambiguous case is omitted.

(8) Though the graphs of the sine and cosine functions are taught, the graph of the tangent function is omitted.

In general, only those aspects of a topic are taught which are necessary to develop relevant skills. Though closure might give a broader understanding of a topic in mathematics, a broader understanding is not necessary to achieve the terminal goals of the course. Furthermore, closure has some negative aspects. It is time-consuming, and it unnecessarily increases the learning complexity for the students.

Problem Solving. Ordinarily, an applied math course implies a heavy emphasis on problem solving. The content we have described includes topics like equation solving, rearranging formulas, and geometric and trigonometric problems, all of which could be called problem solving. However, it does not include many verbal problems, and ordinarily verbal problems are what is meant by problem solving. Verbal problems have not been emphasized because it is difficult to find types which are nontrivial and solvable by the students. The type of verbal problems which appear in traditional math courses are frequently trivial and unrelated to the types of problems which occur in science or technology. The type of verbal problems which do occur in science or technology frequently cannot be solved by the students because their solution presupposes an understanding of scientific principles which the students do not have. Therefore, the goals of the math course have been limited to teaching the manipulative skills and math models which are used in science and technology. The application of math to problems in science or technology has to be delayed until the students learn some scientific and technical principles. The proper place for this application is in the science and technical courses themselves.

Unique Aspects of Each Content Area.

In this section, we will list some of the major unique aspects of each of the five content areas. Though all of the unique aspects cannot be listed, enough will be listed to communicate the overall flavor of the content.

I. ALGEBRA

(1) Given the entry skills of the students, a complete review of fundamental algebra is included. This review includes a heavy emphasis on simple and complicated operations with signed numbers.

(2) The meaning and sensibleness of axioms and principles is demonstrated by the use of numbers.

(3) The manipulation of algebraic expressions and the solution of equations and formulas is based on an understanding of the principles of modern math. This emphasis on principles contrasts with many vocational or technical math textbooks in which rote-mechanical procedures are taught.

(4) Formal strategies for solving equations are an integral part of the instruction, and the strategies used in solving traditional equations are explicitly generalized to the rearrangement of literal equations and formulas.

(5) When various solution-methods for a type of traditional equation exist, that method is taught which generalizes most readily to literal equations and formulas.

(6) The meaning of fractions and operations with fractions are heavily emphasized. Other than the type of numerical fractions which occur in the construction trades (halves, fourths, eighths, etc.) operations with numerical fractions are not emphasized for their own sake. They are used to

show the basic principles which underlie operations with fractions. These principles are then generalized to the types of literal fractions which occur in formulas and derivations.

(7) Other than factoring based on the distributive principle, factoring trinomials and higher polynomials is omitted.

II. CALCULATION

(1) The meaning of numbers and the number system are reviewed.

(2) An attempt is made to develop number sense, which includes both a knowledge of the relative size of numbers and a habit of checking the sensibleness of answers.

(3) Powers of ten and standard notation are treated in great detail.

(4) Direct inspection is used for simple estimations. Decimal-point shifts are used to convert difficult problems to simpler problems in which direct inspection can be used. Powers of ten are used with difficult problems which cannot be converted to simpler ones. (Other methods of estimation which are more "mental" were included in previous years. In general, the students were not fluent enough with numbers and the number system to use these methods effectively.)

(5) Slide rule exercises are designed to systematically cover the full range of possible numbers on each scale. By "systematically" we mean that one-digit, two-digit, three-digit, and four-digit settings are treated sequentially.

(6) Solutions of first-degree equations and formula evaluations which require use of the slide rule are briefly introduced.

III. GRAPHING

(1) The rectangular coordinate system is not restricted to functions in which the variables are "x" and "y". Formulas are also graphed and all concepts are generalized to the graphs of formulas.

(2) Linear and curvilinear graphs are intermingled.

(3) Heavy emphasis is given to the concept of slope and its use in determining relative changes in the values of variables.

(4) In defining slope, the horizontal change (Δh) and the vertical change (Δv) are represented graphically as vectors.

IV. TRIGONOMETRY AND APPLIED GEOMETRY

(1) The general approach to trigonometry is numerical rather than analytical.

(2) Only the sine, cosine, and tangent of angles are initially defined. The definitions of cosecant, secant, and cotangent are not only delayed but deemphasized.

(3) The treatment of interpolation in trigonometric tables is omitted to avoid unnecessary complexity. In calculations, all angles are rounded to the nearest degree.

(4) Instead of using logarithms, the slide rule is used when calculating unknown sides and angles of triangles. Log-trig tables are not used.

(5) The definitions of the trigonometric ratios of general angles are deferred until the trigonometric ratios of reference angles are thoroughly understood.

(6) Obtuse oblique triangles are not introduced until the law of sines and the law of cosines are fully understood in the context of acute oblique triangles.

(7) Trigonometric graphing is restricted to a detailed treatment of the sine wave and its properties with a slight introduction to the cosine wave. The graphs of all other trigonometric functions are omitted.

(8) A numerical approach is used in the treatment of geometric principles and relationships.

(9) The basic properties of circles and half-tangents are reviewed in the context of applied problems in trigonometry.

V. LOGARITHMS AND EXPONENTIALS

(1) The meaning and laws of logarithms are based on a complete treatment of powers of ten, standard notation, and the laws of exponents.

(2) The treatment of negative logarithms is deferred until the treatment of positive logarithms is completed.

(3) Negative logarithms are expressed as single negative numbers. The "-10" form of expressing negative logarithms is neither discussed nor used.

(4) Except for the evaluation of decimal roots and powers, calculations by means of logarithms are not emphasized.

(5) A treatment of interpolation in logarithmic tables is omitted. In finding logarithms and anti-logarithms, values are rounded to the nearest table entry.

(6) Heavy emphasis is given to evaluating and rearranging logarithmic and exponential formulas (especially those involving base "e").

II. LEARNING MATERIALS AND THE USE OF LEARNING PRINCIPLES

The content of the course is communicated by means of programmed materials. A total of 2,922 pages have been written by members of the project staff. The materials are divided into individual booklets for each of the 31 topic-units. In this section, we will discuss the learning materials and the principles of learning which are incorporated in them.

Why Programmed Instruction?

There are mixed feelings about the use of programmed instruction in our educational system. Those who are positive sometimes assume that all human learning is a matter of simple operant conditioning, and they feel that the mere translation of learning materials to the operant conditioning format will guarantee success. This assumption, of course, is somewhat naive. Those who are negative could be so for any of a number of reasons. There are certainly a number of poorly-written programmed texts on the market. When programmed materials are used, the teacher's role is altered and his status is threatened. Also, many educators tacitly assume that the lecture-discussion method is the ultimate of all methods of communication in spite of the fact that little effort is made to justify this assumption by means of objective assessment.

The project staff has never assumed that programmed materials are a self-sufficient method of instruction for the majority of the students. However, with a goal of mastery for average and below-average students, the reasons for using programmed materials are substantial. There are four basic reasons:

(1) Daily personal attention can be provided to each individual student if programmed materials are used.

> Since class time is no longer needed for lecturing, it can be used for assessment and individual tutoring. Therefore, the teacher can interact with individuals and control their individual learning processes. This latter type of control is absolutely essential with average and below-average learners.

(2) Programmed materials put the emphasis on student activity instead of teacher activity.

> Learning is an active process. It occurs only when the student interacts with the learning materials, whether these be in the form of lectures or written materials. There is a much higher probability of this type of interaction if programmed materials are used. The student can proceed at his own pace. The instructional materials are portable, and therefore the student's attention is not required at some definite time or place. And, since each step in his learning process is monitored by feedback, the student is less apt to stop paying attention if he becomes confused.

(3) Programmed materials are a better method of communicating with average and below-average students.

In order to communicate with average and below-average students, great care must be given to the details of the instruction and the success of the instruction. More details can be included in programmed materials than in lectures, and there is a higher probability that the necessary details will be detected since the writer ordinarily analyzes the content much more carefully. Furthermore, the programmed materials can be constantly improved on the basis of feedback from teachers, students, and test results.

(4) Programmed materials are a useful method for coping with absenteeism.

In the ordinary lecture-discussion method, if a student misses a lecture, one segment of the instruction is lost. With programmed instruction, there is no lost segment of the instruction.

General Characteristics of the Programmed Materials.

The programmed materials have been frequently revised during the history of the project. Some of the booklets have been rewritten as many as three times. More effort has been given to revisions of the first-semester booklets since they deal with fundamental skills which must be mastered if the student is to have a chance of going on. In the course of these revisions, the style of the materials has tended to stabilize. The major general characteristics of the materials are listed below:

(1) All of the programming is linear. Though not denying the need for branching, this need is filled by the teacher's tutoring.

(2) The writing has never been restricted by any rigid rules for frame size or number of frames. The size and number of frames is dictated by the particular content.

(3) If there is no non-trivial question to ask in a given frame, no response is required.

(4) All of the instruction is straightforward exposition. There are no discovery exercises.

(5) The materials are broken down into individual booklets covering topic units. Therefore, the student is always confronted with a reasonable goal rather than with a formidable single book containing all of the course content.

(6) Each booklet is broken down into sub-units which are preceded by a verbal orientation and usually followed by a self-test.

(7) Summary frames are introduced at appropriate places.

(8) Detailed strategies and steps for all types of problems are shown.

Reading Skills of Students.

Teachers frequently express apprehension about programmed materials because they fear that their students will encounter reading problems. At the beginning of the project, this fear prompted the staff to avoid the use of technical terms and verbal language as much as possible when writing the materials. However, the anticipated reading problems failed to materialize. And since the avoidance of verbal language in materials is not good instructional practice for many reasons, the materials have become increasingly more verbal in successive revisions. Even with the revised materials, reading problems have not been encountered to any great extent. As a precaution against creating reading problems, the language used is very simple and straightforward, and the statements frequently do not correspond exactly to the precise statements of a professional mathematician. Of course, statements which are mathematically false are avoided.

Reading problems are encountered with many conventional math textbooks because they simply do not communicate with the students. Too many details are skipped, and much of the instruction is either too verbal or too highly related to abstract mathematical stimuli. We feel that reading problems have not been encountered with the programmed materials, even with the inclusion of more verbal language, since the words do not appear in isolation from concrete mathematical stimuli. The following frame from one of the booklets is a good example of the combined use of verbal language and mathematical stimuli:

"Reducing a fraction to lower terms" means "finding an equivalent fraction whose numerator and denominator are smaller." Here is the procedure:

Step 1: We must factor the fraction into two fractions, one of which is an instance of $\frac{n}{n}$.

$$\frac{4}{8} = \frac{(4)(1)}{(4)(2)} = \left(\frac{4}{4}\right)\left(\frac{1}{2}\right)$$

Step 2: We substitute "1" for the instance of $\frac{n}{n}$.

$$\frac{4}{8} = \frac{(4)(1)}{(4)(2)} = \left(\frac{4}{4}\right)\left(\frac{1}{2}\right) = (1)\left(\frac{1}{2}\right)$$

Step 3: We drop the "1" since $(1)(n) = n$.

$$\frac{4}{8} = \frac{(4)(1)}{(4)(2)} = \left(\frac{4}{4}\right)\left(\frac{1}{2}\right) = (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

Complete this reduction to lower terms:

$$\frac{8}{12} = \frac{(2)(4)}{(2)(6)} = \left(\frac{2}{2}\right)\left(\frac{4}{6}\right) = (1)\left(\frac{4}{6}\right) = \underline{\qquad\qquad}$$

Learning Characteristics of Average and Below-Average Students.

When preparing instructional materials, the learning characteristics of the students must be taken into account. Individual differences are too great to permit a precise definition of learning characteristics which fits each average and below-average student. However, there are many characteristics which are evident in a significant number of students of this type. Though undoubtedly incomplete, the following list includes some of the more striking characteristics:

(1) He learns rather slowly and needs a fair amount of practice to master a learning set.

(2) He frequently confuses learning sets which are similar because he does not automatically make all of the discriminations which are necessary for success in learning.

(3) He is frequently unable to understand definitions, principles, or axioms which are communicated in abstract verbal or symbolic language.

(4) His learning is quite specific. That is, he does not frequently transfer a learning set to stimuli which are dissimilar to those specifically used in teaching the learning sets.

(5) He is frequently unable or unwilling to organize what he is doing. This lack of organization is reflected both in a lack of a plan of attack or strategy and in the carelessness of his work.

(6) He does not always encode what he is trying to learn into verbal language, and when he does so, the encoding is frequently too imprecise to accurately control correct overt behavior.

(7) His retention rate, as measured by pure recall, is not high.

The characteristics described above indicate that this type of student is not easy to teach. He has not developed efficient learning habits, and he can easily become confused if any necessary details are omitted from the instruction. Nevertheless, if any instructional materials hope to communicate with him, they must come to grips with him as he is and not as we would like him to be.

Use of Learning Principles.

An attempt has been made to incorporate what is known about the learning process of slower learners in the programmed materials. In the various revisions of the materials, this attempt has been more successful since feedback from the teachers and item analyses of tests have clarified the characteristics of this population of learners. In this section, we will discuss some of the principles of learning which have been incorporated in the materials in an effort to cope with the students' learning characteristics. There are seven sub-sections. Each one contains a description of the learning principles which are used to counteract the seven learning characteristics listed in the last section. The sub-sections are by no means mutually exclusive, and they do not pretend to be an exhaustive summary of the psychology of learning elementary mathematics. Brief as this discussion of learning principles is, it does give some idea of the style of the materials.

(1) Amount of Practice. Since many of the students learn slowly, a fair amount of controlled practice is given when each new learning set is introduced. Ordinarily, a single example-problem would be entirely inadequate. A fair amount of practice is not only necessary for original learning, but hopefully it has some effect on long-range retention.

(2) Discrimination Training. Since the students frequently confuse learning sets which are similar, discrimination frames are used extensively. The purpose of the discrimination frames is to eliminate common errors by forcing the student to contrast the learning sets and to examine more closely the stimuli to which each one applies. The following list contains some of the types of situations in which discrimination frames are used.

(a) Contrasting operations which are frequently confused. For example:

$$3\frac{1}{4} \text{ and } 3\left(\frac{1}{4}\right)$$

$$3 + (-4) \text{ and } 3(-4)$$

$$\sqrt{a^2b^2} \text{ and } \sqrt{a^2 + b^2}$$

(b) Contrasting different orders of operations. For example:

$$ab + c \text{ and } a(b + c)$$

$$a - bc \text{ and } (a - b)c$$

(c) Contrasting the meaning of technical terms. For example:

"terms" and "coefficients"

"reciprocals" and "opposites"

"side opposite" and "side adjacent"

(d) Contrasting an operation with the reverse of that operation. For example:

Just as: $(h + r)t = ht + rt$,
$ht + rt = (h + r)t$

Just as: $1 + \frac{b}{a} = \frac{a}{a} + \frac{b}{a} = \frac{a + b}{a}$

$\frac{a + b}{a} = \frac{a}{a} + \frac{b}{a} = 1 + \frac{b}{a}$

(e) Contrasting the use of axioms. For example, when solving for "b" in the following equations, showing that:

The multiplication axiom is used with: $ab = c$

The addition axiom is used with: $a + b = c$

(3) *Avoidance of abstract verbal or symbolic definitions.* To counteract the fact that abstract definitions, whether verbal or symbolic, frequently do not communicate with this type of student, definitions of this type are avoided. The following statements can be made about the way in which definitions are handled:

(a) *When principles are introduced, they are justified intuitively with numbers.*

For example, the distributive principle:

$$a(b + c) = ab + ac$$

is justified with various sets of numbers like:

$$\begin{array}{lcl} a = 5 & & a = -4 \\ b = 3 & \text{or} & b = -6 \\ c = 2 & & c = -9 \end{array}$$

(b) *When equation axioms are introduced, they are justified intuitively by showing that all derived equations are equivalent to the original equation.*

For example, when introducing the addition axiom, it is shown that all of the following equations have +3 as their root.

$$\begin{aligned} 2x + 5 &= 11 \\ 2x + 5 + 10 &= 11 + 10 \\ 2x + 5 + (-20) &= 11 + (-20) \\ 2x + 5 + (-5) &= 11 + (-5) \end{aligned}$$

(c) *When defining a relationship in geometry, the exclusive use of standard letters with standard position figures is avoided.*

For example, when defining the sine, cosine and tangent of an angle in a right triangle:

(1) Letters other than "a", "b", and "c" are used.
(2) Right triangles in non-standard positions are used.

(4) *Transfer.* To counteract the fact that the slower learner's learning is rather specific, a wide range of transfer is not assumed. In fact, great care is taken to see that each learning set is taught in such a way that it generalizes to all stimuli to which it is meant to apply. The following statements give some idea of the way in which transfer problems are handled:

(a) *A learning set is always explicitly generalized to dissimilar stimuli.*

For example, the distributive principle is explicitly generalized to the following stimuli at the appropriate time.

$$(3)(5)\left(\frac{x}{3} + \frac{1}{5}\right) = (3)(5)\left(\frac{x}{3}\right) + (3)(5)\left(\frac{1}{5}\right)$$

$$(x + 3)(x + 2) = (x + 3)(x) + (x + 3)(2)$$

(b) When a learning set is introduced, it is not assumed that it will generalize to superficially similar stimuli which are really unique cases.

For example, even though a student can solve basic equations like:

$$2x = 10$$
$$-3x = 18$$

it is not assumed that he can solve the following equation:

$$-x = 5$$

(c) A learning set is introduced with enough stimulus diversity to make it generalizable to the set of stimuli to which it must be applied.

For example, when the formula for the area of a triangle is introduced, the discussion is not limited to the standard case in which the base is horizontal and the altitude is vertical.

(d) It is not assumed that the strategies for solving one-letter equations automatically transfer to the rearrangement of functional relationships and literal equations.

For example, when a student knows how to solve for "x" in

$$3x = 7$$

it is not assumed that he can solve for "x" in either

$$3x = y$$

or

$$ax = b$$

(e) It is not assumed that the strategy for graphing traditional "xy" equations automatically transfers to graphing formulas.

For example, when a student can graph

$$xy = 10$$

it is not assumed that he can graph

$$EI = 10$$

(5) Strategies. To counteract the fact that the students frequently do not develop a plan of attack on their own, formal strategies are explicitly included as an integral part of the instruction. Furthermore, pressure is put on the students by the teachers to use the strategies and to lay out all of their steps when doing so. This pressure undoubtedly contributes to the success of the instruction. The following statements can be made about the formal strategies which are taught:

(a) *Shortcuts are avoided.* They are avoided because they frequently lead to negative transfer. They lead to negative transfer because the students cannot discriminate the situations in which they can be used from the situations in which they cannot be used. Here are some examples of shortcuts which are avoided:

(1) *Transposition*, which is replaced by use of the addition axiom.

(2) *Cancelling with fractions*, which is replaced by the use of the two principles: $\frac{n}{n} = 1$ and $n(1) = n$.

(3) *Cross multiplication*, which is replaced by the use of the multiplication axiom to clear the fractions.

Note: If the better students begin to use shortcuts, their use is not discouraged provided that they are not used incorrectly.

(b) *Formal strategies for solving equations are taught, and frames are included in which the student must explicitly state the strategy.* When introducing these strategies, the following points are explicitly discussed:

(1) The general goal is clarified. That is, the general goal is to apply axioms and principles to a complex equation until an equivalent equation which is easy to solve is obtained.

(2) The purpose of each individual step is explicitly stated.

For example: "The addition axiom (adding -5 to both sides) is applied to the following equation in order to eliminate the +5 from the left side."

$$3x + 5 = 13$$

(c) *Whenever possible, only one strategy is taught even when other methods are available.* Teaching more than one strategy is avoided for two reasons: (1) because frequently neither strategy is learned well, and (2) because frequently the students have difficulty discriminating when the alternate methods should be used. Here are some examples:

(1) When solving fractional equations or rearranging fractional formulas, the fractions are always cleared first by using the multiplication axiom.

(2) When solving for a variable which appears under a radical in an equation or formula, the radical is always isolated before the squaring axiom is applied to both sides.

(3) The quadratic formula is taught as a general method for solving quadratic equations. (Note: It is used because the coefficients in quadratic equations which arise in technical work are usually decimal numbers.)

(d) Formal strategies for estimating answers are taught. These strategies include all of the possible stimuli which can be encountered. For example, when estimating quotients, the following strategy is taught:

   (1) If the denominator is a number between 1 and 10, round the numerator and denominator to one digit and perform the short division.

   (2) If the denominator is not a number between 1 and 10, shift the decimal points to make the denominator a number between 1 and 10. Then round and perform the short division.

   (3) If the numerator would become extremely large or extremely small if the decimal points were shifted to obtain a denominator between 1 and 10, do not shift the decimal points. Instead, convert both numerator and denominator as they stand to standard notation and complete the estimation by the standard-notation method.

(e) A higher order strategy covering two learning sets is taught when there is a possibility that the application of the two learning sets can be confused. Here are some examples:

   (1) After the Law of Sines and the Law of Cosines are introduced in the context of solving oblique triangles, the following strategy is taught: try the Law of Sines first; only try the Law of Cosines when the Law of Sines does not work.

   (2) When converting from standard notation to regular numbers and vice versa, the student is taught to think in terms of the relative size of the numbers. Rote rules relating decimal point shifts and exponents are avoided.

(6) Use of Verbal Language. As the project has progressed, the importance of the use of verbal language in mathematics instruction has grown in the eyes of the project staff. By the "use of verbal language," we mean using technical terms and verbally describing operations and strategies. There are various functions which this use of verbal language serves:

(a) It permits easier communication between student and teacher.

(b) It forces the student to discriminate different mathematical stimuli and processes.

(c) It can be used as a cue for recall when the student has forgotten something.

(d) It makes more efficient reviewing possible.

(e) It may improve long-range retention.

One of the functions which verbal language serves deserves special mention. Based on the theory that "thinking" is a process which is either verbal, partially verbal, or at least potentially verbalizable, "thinking" can be conceptualized as an "inner dialogue" which the student has with himself. That is, "thinking" is his ability to verbalize a strategy or plan of attack to himself. For example, when a student is solving for "c" in the following equation:

$$\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$$

he should make statements like the following to himself:

(1) There is one term on the left side and two terms on the right side.

(2) I can clear the fractions by multiplying both sides by "abc".

(3) When multiplying the right side by "abc", $abc\left[\frac{1}{b} + \frac{1}{c}\right]$ is an instance of the distributive principle.

There are obviously many further statements of this type which he should either make to himself or be able to make to himself about his strategy or plan of attack for solving for "c".

Many slower learners are not in the habit of encoding what they learn into verbal language, or at least the encoding which they do is not very precise or useful. Therefore, they do not automatically develop clear strategies for solving equations and problems or, in other words, they do not automatically learn how to "think." Forcing them to verbalize strategies for solving equations and problems is really forcing them to learn how to "think." Hopefully, this effort which the slower learner makes to verbalize what he is doing in mathematics will gradually become habitual and transfer to other courses.

(7) Retention. Though forgetting is a problem with all learners, it is a special problem with slower learners. In order to come to grips with the forgetting which occurs, various features have been built into the materials:

(a) A fair amount of practice is given with the hope that some overlearning will occur.

(b) When a learning set which has not been used recently is needed in the instruction, a review frame is inserted so that recall can be assured.

(c) An attempt is made to give some distributed practice by the sequencing of the booklets. For example, some calculations booklets are inserted in the algebra sequence in the first semester so that the students study algebra at two distinct times in the semester. Furthermore, there is another algebra sequence in the second semester.

III. ASSESSMENT INSTRUMENTS

The criterion for the success of any instructional system is the amount of learning which occurs in the students. To assess the amount of learning which is achieved, the system must be evaluated as objectively as possible. Tests are needed whose items directly measure the attainment of the learning objectives. Since the whole evaluation of the system depends upon the quality of the tests, the test-construction is just as important as decisions about content and the development of learning materials. Therefore, just as the learning materials have been constantly revised, the accompanying tests have been constantly revised and improved.

The general philosophy of testing which has been adopted by the project staff differs from that which frequently exists in conventional classes. In many conventional classes, tests are used as an instrument for determining grades. Since the major goal of the test is to discriminate among the students, the tests are not developed carefully. Many transfer items are included because insufficient effort is given to devise items which assess the attainment of the learning objectives, or at least to assess that which was actually taught. In fact, transfer items are sometimes deliberately inserted because they discriminate better. Unfortunately, tests of this type are not a good instrument for assessing the success of the instruction. Though some of the tests developed by the project staff are used to determine course grades, the general purpose of the tests is diagnostic. The diagnosis is two-fold: (1) diagnosing the success of the system, and (2) diagnosing the success of individual students. The diagnosis of the success of the system is used in order to improve the instructional materials and the general classroom procedure. The diagnosis of the success of individual students is used as a basis for tutoring so that the teachers can compensate for any lack of success of the instructional materials.

General Features of the Assessment Instruments.

A total of 162 tests were administered during the 1968-69 school year. The following types of tests were used:

ENTRY DIAGNOSTIC TESTS
TOPIC POST-TESTS
DAILY CRITERION TESTS
MULTI-TOPIC COMPREHENSIVE TESTS
FINAL EXAMINATIONS

Note: Though topic pre-tests were administered during the first $1\frac{1}{2}$ years of the project, they were not used during the 1968-69 school year.

Before discussing the content and purpose of the various types of tests, we will list some general features which characterize the whole effort at assessment:

(1) A test is given every day. Daily testing provides a constant detailed assessment of the progress of each student and the success of the instructional system. Though some of the tests do not count towards a course grade, all of them are used as a basis for tutoring and as a source of information about needed modifications in the learning materials.

(2) All of the items require a constructed response. Since the tests are used as a basis for tutoring, a "constructed-response" format is used so that the student's work can be examined. The "true-false," "multiple-choice" and "matching" formats are not used because they do not provide a written record of the student's work.

(3) The tests are designed for rapid correcting. To make the correcting of the tests as easy and rapid as possible, numbered boxes are provided in a column in the righthand margin of each page. The students are required to record their answers in these boxes.

(4) Transfer items are generally avoided. Since the items are designed to assess the attainment of the learning objectives, items which do not directly test the stated objectives are avoided.

(5) No partial credit is given for any item on a "graded" test. Even when an item is complex or requires some calculation, it is graded as either "perfectly correct" or "incorrect." Though this standard is a fairly stringent one for the students, it is maintained in order to force the students to learn to be careful and accurate.

(6) "Graded" tests are scored on a strict percentage basis. 70% is considered a passing grade, even though retests are sometimes required for students who achieve higher scores.

(7) When retests are required, they are not used to adjust the student's grade. Retests are used after tutoring only as an assurance that the student has actually mastered the learning objectives.

Specific Features of Each Type of Test.

In this section, we will describe the specific features of each type of test. The specific features include its content and purpose, and whether it is used for grading or not. Though topic pre-tests were not used during the 1968-69 school year, their specific features will also be included.

Entry Diagnostic Tests. Two diagnostic tests are given at the beginning of the course. These two tests, entitled "Pre-Test: Arithmetic" and "Pre-Test: Algebra," were described earlier in the section on the characteristics of the entering student. Copies of the two tests are given in Appendices B-1 and C-1. Each test is designed for administration in one 50-minute period. The tests measure each student's entry behavior in terms of skills in basic arithmetic and basic algebra. Since most of the items test learning objectives which are directly reviewed in the course itself, the results are not used to determine specific remedial programs for individual students. Though the results are mainly used to characterize the general entry level of the students, they do serve the following specific functions:

(1) Both tests quickly characterize individual students for the teachers.

(2) Both tests are used later in the course as multi-topic comprehensive tests, and therefore pre-test and post-test scores are available for a segment of the instruction.

(3) The "Pre-Test: Algebra" is used to determine assignment to a "Special" group which will be discussed later.

Topic Post-Tests. The two-semester course is divided into 31 topic-units for which individual programmed booklets have been written. The purpose of the "topic post-test" is to assess the attainment of the terminal learning objectives in each topic-unit. A copy of the topic post-test for Algebra IX: Formula Rearrangement is given in Appendix E-1. A copy of the topic post-test for Logarithms II: Common and Natural Logarithms is given in Appendix E-3. Since these tests are designed to assess terminal learning objectives, items which test subsidiary learning objectives are generally not included. The tests are given immediately after the completion of a topic-unit. A full 50-minute period is allowed for each test of this type.

Since the scores on the topic post-tests are used in the determination of student grades, three parallel forms of each test have been prepared as a precaution against cheating. By "parallel forms," we mean that the tests contain parallel items. The three items below are "parallel," for example, since they test the same learning objective and have the same difficulty level:

If $3x = 7, x = ?$
If $2y = 5, y = ?$
If $4p = 9, p = ?$

The parallel forms are also used for the retesting which is described below:

Topic post-tests are examined by each student the day after the test is given. All students are required to rework any items which they had wrong. A lowest acceptable score on each topic post-test is determined by the teachers. This lowest acceptable score depends on how difficult the test is and how essential the learning objectives in the test are. Ordinarily, the lowest acceptable score is around 80%, although 90% is required on some tests, and 70% is accepted on others. Any student who does not achieve the lowest acceptable score is required to take a retest after being tutored by the teacher. One of the other two parallel forms is used for the retest. Though retests are mandatory, retest scores are not used in the determination of student grades. Only the score on the initial test is used in determining grades.

Topic Pre-Tests. Topic pre-tests were administered only during the first one and one-half years of the project. The purpose of a topic pre-test is to measure a student's entry behavior before the instruction for a particular topic-unit begins. When they were administered, each pre-test was a parallel form of the post-test for that topic-unit. A comparison of pre-test and post-test scores was used to measure the learning gains. The topic pre-tests were used as a more detailed assessment of the entry behavior of the students. Since all but the first pre-test of this type were given after some instruction had occurred, they also served as a measure of any general recall of topics which resulted from instruction on other topics. They were given in order to assure the staff that the instruction was not beginning at too elementary a level. Since the student characteristics do not change much from year to year, the use of pre-tests has been discontinued so that the class time absorbed by them can be used more profitably for instruction and tutoring.

Daily Criterion Tests. The content in each topic-unit is covered in an average of three or four daily assignments. A "daily criterion test" is written for each daily assignment. Copies of the four daily criterion tests for Algebra IX: Formula Rearrangement are given in Appendix E-2. Copies of the daily criterion tests for Logarithms II: Common and Natural Logarithms are given in Appendix E-4. The purpose of these tests is two-fold: (1) to serve as a check that the students have completed the assignment, and (2) to serve as a basis for tutoring any students who have not mastered the learning objectives for that day. Since these tests are a pure diagnostic tool, they are not graded. And since their goal is diagnostic, the items assess both terminal and subsidiary learning objectives. The daily criterion tests are administered at the beginning of each class period. They are limited to one side of a single sheet of paper, and ordinarily, they can be completed in 15 or 20 minutes. As soon as each student completes this test, it is corrected in class by the teacher. If tutoring is required, the tutoring is accomplished during that class period.

Multi-Topic Comprehensive Tests. During the 1968-69 school year, a decision was made to include some multi-topic comprehensive tests in the last half of the second semester. These tests were used as a method of reviewing most of the major terminal objectives of both semesters. The staff felt that such a review was necessary in order to come to grips with the forgetting which occurs. The following five tests were used:

Arithmetic (Parallel form of Pre-Test: Arithmetic)
Basic Algebra (Parallel form of Pre-Test: Algebra)
Intermediate Algebra
Graphing
Trigonometry

Copies of the "arithmetic" and "basic algebra" tests are given in Appendices B-1 and C-1. Copies of the "intermediate algebra," "graphing," and "trigonometry" tests are given in Appendices I-1, J-1, and K-1. These tests are designed for administration in one 50-minute period. They are used as a basis for remedial work with those students who exhibit a significant amount of forgetting. In determining a student's second-semester grade, each comprehensive test is counted as much as a single topic post-test.

Final Examinations. Copies of the final exams for Technical Mathematics 1 and Technical Mathematics 2 are given in Appendices G-1 and H-1. The final examination for each semester includes items which cover the various topics in that course. Since the tests are designed for administration in 1 hour and 45 minutes (with 2 hours usually allowed as the maximum), only a sampling of the terminal objectives for each topic is possible. When this sampling is done, care is taken to incorporate the most important terminal objectives for each topic. The student's score on this test counts for one-third of his final grade.

IV. CLASSROOM PROCEDURE

During the four-year history of the project, the classroom procedure has gradually evolved into the total use of a learning center with different treatments for three different ability-levels of students. This evolution into the use of a learning center for the whole operation was not anticipated, and changes in this innovative direction were made cautiously because the project was simultaneously responsible for the instruction of large numbers of students. Since the physical facilities have also had an effect on the evolution of the classroom procedure, a brief description of these facilities will be given first.

Physical Facilities at MATC.

The Milwaukee Area Technical College is the largest school of its kind in the United States. It includes all of the following divisions: a junior college, a technical college, an adult vocational school, an adult high school, an apprentice school, and a continuation high school for students under 18 years of age. Including both full-time and part-time students, more than 35,000 were enrolled during the 1968-69 school year. Students enroll in either the day division or the evening division. Though facilities are used in other parts of the city, the major campus is situated in downtown Milwaukee. The campus encompasses three square blocks, with one major multi-level building on each of the three blocks.

The mathematics project is headquartered in a separate small building on the downtown campus. This building was purchased by the school from a movie film distributing firm. Since it is old and scheduled for razing in a few years, it has never been renovated into classrooms. For the first two and one-half years of the project, classes were conducted in ordinary classrooms scattered throughout two of the three major buildings. During this time, both the classrooms and the teachers' offices were in buildings which are physically remote from the Project Center, and communication between the teachers and the project staff was not accomplished easily. If more concentrated facilities had been available from the start, the development of a learning center would probably have occurred sooner.

Goals in the Development of a Classroom Procedure.

In the development of a classroom procedure, the staff had three general goals: (1) a procedure which maximized the amount of learning in the students, (2) a procedure which could be inserted into the ongoing operation and scheduling of a large educational institution, and (3) a procedure which had a chance of long-range survival in an actual school setting. Each of these three goals will be discussed separately.

Maximizing the amount of learning has always been the primary criterion in the evolution of the classroom procedure. In order to maximize the amount of learning, a procedure was needed which included the following aspects:

(1) Personal attention for each student according to his needs, in spite of the large number of students.

(2) Control over each student's effort and attendance.

(3) Flexibility in handling absentees, retests, and assignments to special classes.

(4) An attitude of responsibility for the students on the part of the teachers.

Any decision about the classroom procedure always had to be made in terms of what is feasible in a large educational institution. That is, such things as the availability of manpower and physical space and the limits on the flexibility of scheduling had to be taken into account. Therefore, the development of a classroom procedure was not as unrestricted as it might have been. But since the restrictions encountered are characteristic of those in almost any actual school setting, the procedure which has been developed is clearly a realistic one.

As the classroom procedure has evolved, the staff has been very concerned with its potential for survival. This concern has been prompted by the fear that the product of four years of intensive effort might easily be abandoned for any of a number of reasons. Some of the major concerns have been:

(1) Reasonable limits on the expense of the system.

(2) Reasonable limits on the amount of effort required from the teachers.

(3) A limitation of teacher activities to strictly professional duties.

(4) An increasing involvement of the teachers in the contribution of ideas for the improvement of the system.

We have been especially concerned with the attitude of the teachers since any system which they do not like will be short-lived.

In the rest of this section, we will give a historical resume of the evolution of the classroom procedure into the use of a learning center. The procedure used in each semester during the past four years will be described and criticized. The criticisms will justify the changes which have been made. Though this resume is somewhat long, it will provide an insight into the staff's attempt to be flexible and to profit from experience. For any reader who is interested, the empirical results for each semester are given in Chapter 3.

1965-66 (Technical Mathematics 1 - Pilot Classes).

A decision was made to experiment with three pilot classes during the fall semester of this year. Though this experimentation put a tremendous burden on the project staff, since the project did not officially begin until June, 1965, the staff felt that it was necessary before attempting the instruction of a large number of students during the 1966-67 school year. A total of 73 students were assigned to the three classes. The three teachers who conducted the classes were also involved in the preparation of learning materials and tests.

Each pilot class met 4 days per week with one 50-minute period per day. The course was divided into 11 topic-units. After some experimenting during the first five topic-units, including five televised lectures on the rudiments of slide-rule operations, the following classroom procedure was adopted for each topic-unit:

(1) The instruction was accomplished by means of daily assignments in the programmed materials prepared by the staff. No formal lectures were given. Even group discussion was generally discontinued so that class time could be devoted to interactions with individual students.

(2) Parallel pre-tests and post-tests were administered for each topic-unit.

(3) Daily criterion tests were administered in order to assess each daily assignment. These tests were taken at the beginning of each class and were immediately corrected by the teacher. They were used both as a check to see that the assignment had been done and as a basis for tutoring.

(4) If the tutoring could not be completed during class time, students were assigned to report to the Project Center at some time before the next class meeting. This tutoring, however, had to be kept at a minimum because the teachers were also responsible for the preparation of instructional materials and tests.

(5) Considerable pressure was exerted on the students both to attend class and to complete the daily assignments. Absentees were brought current by their respective teachers.

The following comments are based on the experience with the pilot groups:

(1) The teachers were, in general, satisfied that the classroom procedure would work with large numbers of students and sections.

(2) A decision was made to abandon the use of television. The use of televised lectures greatly increased the amount of time needed to cover slide-rule operations, and there was considerable difficulty in rescheduling these lectures for students who were absent. Furthermore, the production of the video tapes was expensive, and they seemed to have no unique advantage since the content presented on television could also be presented in programmed materials.

The spring semester and summer of 1966 were used by project personnel in preparation for teaching all entering technical students in the fall semester of that year. Aside from the revision of materials, the following specific preparations were made:

(1) Learning carrels equipped with rear screen slide projectors (Kodak Carousel 20mm Slide Projectors) were built at the Project Center, and exercises for improving skills in operations with signed numbers, estimation, and slide rule scale reading were prepared.

(2) A one-week in-service training program was conducted for the six teachers assigned to handle the Technical Mathematics sections. Five of the six teachers were new to the faculty; they were hired specifically to handle the math classes in conjunction with the project.

1966-67 (Technical Mathematics 1).

From this point on, the project was responsible for the instruction of all entering technical students. The 503 students who entered in September, 1966, were assigned to one of 24 sections which met 4 days per week (one 50-minute period per day) in a conventional classroom setting. In terms of a full teaching load, six teachers were needed to handle the 24 sections.

In general, the classroom procedure developed with the pilot groups was continued. For each of the topic-units, this procedure included:

(1) A parallel pre-test and post-test.

(2) Daily criterion tests used as a basis for tutoring in the classroom.

(3) Tutoring outside the classroom for students who needed further help.

Pressure was exerted on all students to attend the classes and complete the daily assignments. Each teacher was responsible for the make-up work of absentees. Two new features were included:

(1) If a student did not attain a satisfactory score (usually 80%) on a topic post-test, he was required to take a retest after being tutored by his teacher. Retests were taken outside of class time. They were scheduled and administered by the individual teachers. The parallel pre-test was used for this retesting.

(2) When necessary, teachers assigned some students to the Project Center to work on the exercises (signed numbers, estimation, slide rule scale reading) on the learning carrels. This work was done outside of class time.

The following comments are based on this first experience with teaching large numbers of students:

(1) The procedure, in general, worked reasonably well.

(2) The teachers found it difficult to do make-up work with absentees during the regular class period. They also found it difficult to schedule and administer retests after a topic post-test, since their schedules frequently conflicted with the schedules of the students.

(3) The number of students who used the exercises on the learning carrels was too small to warrant the development of further materials of this type. The staff concluded that the time and expense involved could be better used elsewhere.

1966-67 (Technical Mathematics 2).

Although the grant proposal did not include the development of a system of instruction for Technical Mathematics 2, a decision was made during the first semester to continue with the same system of instruction during the second semester. Therefore, ten new programmed booklets with their accompanying tests were prepared.

The 303 students were assigned to one of 14 sections which met 4 days per week in a conventional classroom setting. In terms of a full teaching load, $4\frac{1}{2}$ teachers were needed, with $3\frac{1}{2}$ handling the 14 sections and 1 handling the services (described below) provided by the Project Center.

The classroom procedure was similar to that used in Technical Mathematics 1 with the following changes:

(1) A decision was made to eliminate the pre-tests for topic-units. Anticipating that the pre-test scores would be very low because most of the second-semester topics were new to the students, the staff felt that the class time needed for them should be used for instruction.

(2) The Project Center was used more extensively in order to make the procedure function as efficiently as possible. It was used to provide the following services:

(a) Absentees. Exerting pressure on the students to attend class regularly sometimes fostered an unpleasant student-teacher relationship. Furthermore, it was difficult for a teacher to administer make-up work while performing his regular functions in the classroom. Therefore, a decision was made to handle absentees in the Project Center. Students were informed to report to the Project Center after any absence. They could report at any time during the day. At the Project Center, the absentees were brought current before being reinserted into their regular classes. The explanation for each absence was discussed and recorded at the Project Center.

(b) Retests. Teachers found it difficult to schedule and administer retests after tutoring students who achieved an unsatisfactory score on a topic post-test. Therefore, a decision was made to administer these retests in the Project Center. The individual teachers were still responsible, however, for the remedial tutoring before the retest.

(c) Special class for fast learners. Teachers had reported that many "A" and "B" students profited little from the daily criterion tests, since they understood the material after completing the assignment in the programmed booklet. Yet if these faster learners remained in the regular classes, their daily tests had to be corrected by the teacher, and consequently he had less time for the slower learners who needed tutoring. As an incentive for the faster learner, and as an attempt to maximize the efficient use of the teacher's time, a decision was made to assign faster learners to the Project Center. This assignment was made by the teachers.

Though still paced by the ordinary schedule of the course, the burden of learning was placed more directly on the faster learners themselves. They took all daily criterion tests at the Project Center at their own convenience, with the stipulation that they had to complete the daily tests at least the day before the assigned date for the post-test. Ordinarily, these students corrected their own daily tests by means of posted answer keys. Post-tests were taken by them on the assigned date at the Project Center. Any student requiring much tutoring or achieving an unsatisfactory score on a post-test was returned to his regular class and the regular classroom procedure. New students were also assigned to this special class during the semester.

The following general comments can be made about the results in Technical Mathematics 2:

(1) Though the topics covered were clearly more difficult, the method of instruction worked reasonably well.

(2) The experiment with a special class for faster learners was successful. Though students were assigned to this group with some caution at first, approximately 20% of the students were in the group by the end of the semester. The students who were assigned to it liked the freedom which it provided. There was no significant decrease in their achievement levels. Furthermore, more teacher-time was made available for the slower students.

(3) The teachers complained that some parts of the materials were too sketchy, and that the length of some assignments was too long. In either case, the students not only took a long time in completing the daily tests, but they also made many errors on them. Therefore, the teachers found it difficult to complete the necessary tutoring during the available class time.

1967-68 (Technical Mathematics 1).

The 478 students who entered in September, 1967, were assigned to one of 20 sections. In terms of a full teacher-load, 6 teachers were needed, with 5 handling the 20 sections and 1 providing the services in the Project Center.

The classroom procedure was similar to that used in Technical Mathematics 2 in the previous year with these exceptions:

(1) The number of class days per week was increased from 4 to 5 without increasing the number of credit hours for the course. This increase was made so that the length of the daily assignments could be shortened. The staff felt that shorter daily assignments would reduce the amount of required tutoring.

(2) The pre-tests for topic-units were eliminated from Technical Mathematics 1. Since the characteristics of the students do not change much from year to year, the staff felt that they now had a fairly objective assessment of the entry skills of the students. Furthermore, they felt that the class time previously used for pre-tests could be more profitably used for instruction.

(3) The "special class for fast learners" was included for the first time in Technical Mathematics 1. Approximately 20% of the students were assigned to this group on the basis of their scores on a pre-test in algebra.

(4) Besides handling absentees, retests, and the special class for fast learners, the Project Center also provided tutoring service for slower learners who needed an extraordinary amount of attention. For some of these students, the pace of the course was slowed down.

Though the use of a Project Center with satellite classrooms worked reasonably well, there were continual problems due to the fact that the classrooms and offices of the teachers were in a different building than the Project Center. The following difficulties and inefficiencies were obvious:

(1) Transporting materials from the Project Center to the satellite classrooms and vice versa was a continual problem. Books and tests had to be transported to the classrooms, and the post-tests had to be returned for analyses.

(2) The teachers had a constant problem with the scheduling of tutoring sessions outside of class time. Their schedules frequently did not coincide with the schedules of the students.

(3) Shifting students from the regular classes to the special class for faster learners and vice versa caused some logistics problems. An elaborate card system and frequent phone calls were necessary in order to make sure that each student was accounted for.

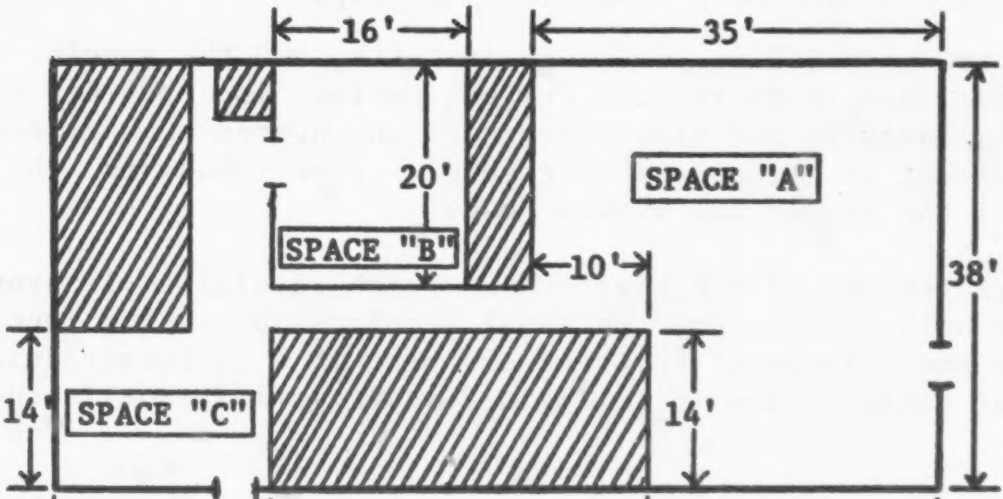
(4) Though some attempt had been made to combine sections with small numbers of students, it was difficult to reassign students to new sections so that teaching manpower was used efficiently.

(5) Since the offices of the teachers and the classrooms were in a different building from the Project Center, the teachers were relatively isolated from the Project Center. This isolation forced them to keep a duplicate set of records, and it prevented quick and easy communication between the teachers and the other members of the project staff.

1967-68 (Technical Mathematics 2).

In order to remedy some of the difficulties and inefficiencies involved in the use of satellite classrooms, a decision was made to use the Project Center as a learning center for all aspects of the instruction during this semester. Though not designed for instructional purposes, enough space was available in the building to handle the students. A diagram of the floor plan of the Project Center is given below:

FLOOR PLAN
MATHEMATICS LEARNING CENTER
MILWAUKEE AREA TECHNICAL COLLEGE



Space "A": Main Classroom (Capacity = 62 students)
Space "B": Special Classroom (Capacity = 22 students)
Space "C": Study Classroom (Capacity = 18 students)

Note: Shaded areas are lavatories and storage space.

Three spaces were available for instruction. The use of each space is described below:

Space "A": This is the main classroom area. Enough learning booths were constructed in it for 62 students. This classroom absorbed the function of the various satellite classrooms, except that the various smaller classroom sections were combined into one large section. Usually two teachers were present during any period. In order to ease the burden of handling a large number of students in one concentrated space, all work with absentees, the tutoring for retests, and the administration of retests was done elsewhere.

Space "B": This small classroom was used for a variety of functions:

(1) The make-up work of absentees.
(2) Tutoring for retests and administering them.
(3) Supervising the special class for faster learners.
(4) Doing some remedial work with new students at the beginning of Technical Mathematics 2.
(5) Supervising some of the very slow students on an individual basis.
(6) Supervising some students who wanted to take Technical Mathematics 1 during the spring semester.

This classroom was manned by one teacher per period. Since he had such a variety of functions to perform, an assignment to this classroom was more difficult than an assignment to the main classroom.

Space "C": This space was used as a study area for the special class of faster learners. They entered and left by a special door so that the traffic in the main classroom could be kept at a minimum. The keys for the daily tests were posted on a bulletin board in this area so that the "specials" could correct their own tests.

It should be obvious from the description of the use of the spaces that a type of team teaching was used. Students received various services from different teachers. For example, though the student in the regular class ordinarily was under the control of the teacher or teachers in Space "A", he reported to a different teacher in Space "B" when absent or when required to take a retest.

The 282 students were assigned to report to the Learning Center during one of 7 periods. In terms of a full-time load, $4\frac{1}{2}$ teachers were used to man the Learning Center. One teacher was assigned to Space "B" during each period, and ordinarily two teachers were assigned to the regular class in Space "A".

The decision to concentrate all aspects of the instruction in a Learning Center had an immediate effect on the administrative efficiency of the instructional system. The need to transport learning materials to remote classrooms disappeared. The logistics of shifting students from one period to another or from the "regular" to the "special" class was simplified. Communication between the teachers and the project staff was accomplished quite easily.

Besides these general improvements in administrative economy, there were definite benefits both for the teachers and the students. Some of the major benefits for the teachers were:

(1) Because of their close contact with members of the project staff, they could take an increasing role in the decisions made about the learning system.

(2) Each teacher's contact with students was limited to his 4 regularly-assigned periods per day. Since all "outside" tutoring was handled by the teachers assigned to Space "B", the tutoring formerly performed by the teachers in their "free" periods was eliminated.

(3) They were relieved of much of the record-keeping since this could be done by clerical help.

Some of the additional benefits for the students were:

(1) Continual tutoring in the Learning Center was available from 7:30 a.m. to 4:30 p.m. daily. Formerly this "outside" tutoring was available only during their teacher's free periods.

(2) Some of the slower learners were handled on a more individual basis by the teacher in Space "B".

(3) Special diagnostic and remedial service was provided for new students enrolling in Technical Mathematics 2.

(4) Various students were allowed to take Technical Mathematics 1 on an individual basis even though that course was not officially scheduled for the spring semester.

In general, this first attempt to handle all aspects of the instruction from a Learning Center was successful. The students were offered a very flexible system in spite of the fact that the teachers had a reduction in their student-contact hours and their non-professional functions. However, the following difficulties were encountered:

(1) The Learning Center was overmanned with teachers.

(2) When two teachers shared the duties in the regular classroom (Space "A"), both teachers were somewhat hesitant to assume control, and consequently the classroom control suffered.

(3) With the frequent shifting of students from one room and teacher to another for various functions, it was not always clear to the students who represented the final authority for them.

(4) Providing special services such as a slower rate for slower learners, taking the course on an individual basis, etc. led to some difficulties. Though ideal for the students, these extra services absorbed a great deal of the teachers' time even though they were provided to only a small number of students. For what was accomplished, this inefficient use of teacher-time was questionable.

1968-69 (Technical Mathematics 1 and 2).

The Learning Center was again used for all aspects of the instruction during both semesters. Basically the same system was used with the following innovations:

(1) Teacher aides were used. The aides, who were carefully selected from second-year technical students, were paid at an hourly rate of $2.00. They were used during both semesters. They performed various non-professional functions like organizing materials and taking attendance. They also performed some professional functions like test-correcting and tutoring. All functions of the aides were directed by the teachers. The use of teacher aides was attempted for the following reasons:

   (a) To eliminate as much as possible the need to have more than one teacher in a room at a time.

   (b) To provide more manpower at a lower cost by reducing the number of required teachers.

   (c) To give more flexibility in the use of manpower by having as large a supply of manpower as possible.

(2) Some students were transferred to a Junior College developmental math course during the first few months of Technical Mathematics 1. These students were extremely slow learners who could not keep up with the pace of the course. In the developmental math course, which also uses the materials prepared for the Technical Mathematics course, these students could proceed at their own rate.

(3) A formal class for slower learners was begun approximately halfway through Technical Mathematics 2. Though some attempt had been made earlier to come to grips with these problem learners on a more individual basis, a formal class for them was never offered. The number of students in these classes ranged from 5 to 10 per period. Though handled in smaller groups, these students were required to maintain the pace of the regular students.

(4) Comprehensive exams were given during the last half of Technical Mathematics 2. Time was provided for tutoring after these exams, and students who received low scores were required to take retests. The comprehensive exams were inserted in an effort to force the students to integrate the topics in the course and to have one final review as a means of increasing their long-range retention.

The 479 students in Technical Mathematics 1 were assigned to one of 7 periods; the 263 students in Technical Mathematics 2 were assigned to one of 6 periods. In terms of a full-time load, $4\frac{1}{2}$ teachers were needed during the first semester and $3\frac{1}{4}$ teachers were needed during the second semester.

Though the harmony of the year was disrupted by a 41 day teachers' strike, which included the last two class weeks and exam period of the first semester and the first three class weeks of the second semester, the results were generally satisfactory. Here is a brief assessment of the three innovations:

(1) Teacher Aides. The number of aides per period depended upon the number of students assigned to that period. During Technical Mathematics 1, there were either 1 or 2 aides per period. During Technical Mathematics 2, there were no aides in some periods and 1 aide in the larger sections. Considering the fact that aides were used for the first time and there was some ambiguity about the functions they should perform, the use of aides was successful.

(2) Transfers to the Developmental Math Course. Only 8 students were transferred to the developmental math course. These extremely slow learners were easy to identify. They could not cope with the pace of the Technical Mathematics course. In fact, the teachers in the developmental math program reported that these students were even slow when compared to the students in that program. Ordinarily, these students are eventually counselled into some other program.

(3) Class for Slower Learners. The formal class for slower learners was not begun until the middle of the second semester. At that time, a small number (between 5 and 10 per period) of students were assigned to the teacher in Space "B". Though still responsible for absentees, retests, etc., this teacher devoted as much time as possible to the slower students. When dealing with these students, the teachers put a great emphasis on forcing them to lay out all steps in every problem. As much time as possible was devoted to requiring these students to verbalize what they were doing in each step and why they were going it. This special attention for the slower students did lead to significant gains in their test scores. The teachers felt, however, that a better job could have been done if they had not been responsible for other functions at the same time. They also felt that if this treatment for slower learners were begun at the beginning of the first semester, a higher percentage of the slower learners could successfully complete both semesters at the required pace of the course.

(4) Comprehensive Exams. These exams seemed to serve a useful function. They made the students realize that the retention of content is important. They also forced the students to integrate material which was learned at various times during the year. The exams were given without any formal review in order to get as uncontaminated a measure of retention as possible. Undoubtedly, some formal review would have raised the scores on the comprehensive exams. In terms of contributing to the students' long-range retention, however, it is an empirical question whether the review should precede or follow the exams.

Though the results during this semester were satisfactory, one general problem persisted. The team approach to teaching, in which a given student received different services from two different teachers in two different physical locations, prevented his identifying with one teacher. This division of authority was further compounded when more than one teacher was assigned to the Main Classroom. The lack of a one student - one teacher relationship was bad for some students, and it was also bad for the morale of the teachers since there were no students for whom they felt completely responsible.

Projected Changes for 1969-1970.

The following changes will be made in the classroom procedure during the 1969-1970 school year:

(1) In addition to the regular class and the class for fast learners, a class for slow learners will be set up at the beginning of the first semester. Students in each period will be assigned to one of these three classes on the basis of their scores on a diagnostic algebra test. During the year, students will be shifted from one class to another on the basis of their actual performance. Though some experimenting will be necessary to determine the optimal number of students in the class for slower learners, the tentative plans are to limit this number to 6 or 8 at a time. The teacher responsible for this class will decide whether a student should be sent to the developmental math class or retained in the Technical Mathematics course. Only those students will be retained in Technical Mathematics who can reasonably keep up with the pace of the course.

(2) An attempt will be made to blend the student-teacher relationship of the ordinary classroom with the flexibility of a Learning Center. Only two teachers will be assigned to each period. One teacher will be responsible for the regular class and the class for fast learners, and one teacher will be responsible for the class for slow learners. Though a student may be shifted from one class to another during the year, at any one time only one teacher will be responsible for his ordinary class work, make-up work, and post-test remedial work. Since the single teacher who is responsible for both the regular class and the class for fast learners will be responsible for the vast majority of the students, he will be assisted by two teacher aides. At least tentatively, one of these aides will handle the class for fast learners and the make-up work of absentees. The other aide will help the teacher in the regular classroom.

(3) A tentative decision has been made to abandon the use of retests after topic-unit tests. The teachers feel that this type of retesting is too time-consuming, and that the same effect can be accomplished by requiring the student to rework only those items on which he makes an error. They also feel that a total retest, including those items which a student has correct, has somewhat the flavor of a punishment.

(4) The attempt to handle special cases on an extremely flexible basis will be abandoned. By "special cases" we are referring to students who want to take Technical Mathematics 1 during the second semester, students who want to begin Technical Mathematics 1 when a significant part of the first semester is over, and so on. Though the staff has always attempted to service students of this type, doing so has proven to be very inefficient. In fact, that type of flexibility consumes more teacher time than it is worth.

(5) Since only two teachers will be assigned to each of 7 periods, only $3\frac{1}{2}$ teachers will be needed to handle the estimated 500 students in Technical Mathematics 1. Even including the cost of aides and clerical help, this reduction in teaching manpower represents a substantial saving.

(6) Though the passing grade for Technical Mathematics 2 will remain at 70%, the passing grade for Technical Mathematics 1 will be raised to 75%. This change will be made because students who pass Technical Mathematics 1 with a grade between 70% and 75% rarely, if ever, pass Technical Mathematics 2. And in spite of the fact that they do not pass Technical Mathematics 2, they require a tremendous amount of tutoring during the second semester.

General Trends in the Development of the Classroom Procedure.

Though many changes have been made in the classroom procedure during the past four years, some of the major changes deserve special mention. These major changes will be listed and briefly discussed in this section.

Use of a Learning Center. During the four years, the classroom procedure has evolved from (1) the use of ordinary classrooms to (2) the use of ordinary classrooms with auxiliary functions performed in a Learning Center, to (3) the complete use of a Learning Center with team-teaching, to (4) the use of a Learning Center which includes the one teacher - one student relationship of the ordinary classroom. The use of a Learning Center has given the system of instruction a flexibility and administrative economy which is impossible when many scattered classrooms are used. It has relieved the teacher of many student-contact hours aside from his regularly scheduled class time. It has made possible such elements as separate classes for the fast, regular, and slow student, the availability of tutoring throughout the day, an easy method of handling absentees and retests. It has also made possible the use of various types of para-professional help. The one teacher - one student relationship of the ordinary classroom is being reestablished because it is more satisfying to both the teachers and the students, and it provides a better method for controlling the progress of each student.

Control of Student Learning. A major goal of the project has been an attempt to gain control over the learning process and effort of each student. This control is maintained by various mechanisms such as daily tests and tutoring, minimum required scores on unit-tests, and a method for handling the make-up work of absentees. The level of performance of the students is clearly affected by the willingness of the teachers to use these mechanisms as a means of taking control of each student's effort and progress.

Role of the Teacher. In the system of instruction which has been developed, the role of the teacher is different than the role of a teacher in the traditional self-contained classroom. He does little, if any, lecturing. Other than suggestions he makes, which may or may not be followed, he has little control over the course content, instructional materials, and tests. He even has little control over course grades which are assigned on a percentage basis. Furthermore, since test scores are analyzed in terms of the various sections, there is a certain amount of implicit assessment of the success of the teachers themselves. Probably the best description of the role of the teacher is that he is a "manager of learning." His major responsibility is to gain control over the learning process of each student and to contribute to that learning process when necessary. He makes decisions about the class (fast, regular, or slow) in which each student will be handled. He supervises the teacher aides. He does some test correction. He makes constructive criticisms about any element in the instruction system.

Separate Treatments for Different Ability-Levels. The classroom procedure has been designed to offer personal attention to the students according to their needs. Therefore, during any given period, separate classes and treatments are offered to fast learners, regular learners, and slow learners. Even though all three groups proceed through the materials at the same pace, the personal attention for fast learners has been reduced to a minimum so that a teacher can be freed to devote a maximum amount of personal attention to the slow learners.

Use of Para-Professional Personnel. There has been an increasing use of teacher aides and clerical help. This use of para-professional personnel has increased the amount of teaching manpower with a decrease in cost in spite of the fact that the level of performance of the students has not suffered. Furthermore, the teachers have been relieved of many non-professional duties so that more of their time can be directly devoted to interactions with individual students.

Reduction in Cost. Though a reduction in the cost of teaching has never been a primary goal of the project, nevertheless, such a reduction has occurred. During the past three years, the number of required full-time teachers has been reduced from 6 to $3\frac{1}{2}$ for Technical Mathematics 1 and from $4\frac{1}{2}$ to 3 for Technical Mathematics 2, even though the number of class meetings per week has increased from 4 to 5. The number of teachers now needed is considerably less than would be needed if the course were taught conventionally. Though some of the money saved by the decreased need for higher-priced teaching manpower is used to pay teacher aides and clerical help, there is still an overall saving.

CHAPTER 3

RESULTS AND DISCUSSION

TECHNICAL MATHEMATICS CLASSES AT MILWAUKEE AREA TECHNICAL COLLEGE

The data contained in this chapter is crucial. Since the primary goal of any system of instruction is to make learning occur, the success of a system of instruction must be measured in terms of the achievement of learning objectives. That is, the success can only be measured in terms of objective, empirical facts. The results will be reported and discussed under the following six major headings:

(1) Pilot groups in 1965 (including a comparison with conventional classes during that year)

(2) Technical Mathematics 1 (1966, 1967, 1968)

(3) Technical Mathematics 2 (1967, 1968, 1969)

(4) Comprehensive Exams (1969)

(5) Student Attitude Questionnaires (1967, 1969)

(6) General Discussion of Results at MATC

Pilot Groups (1965)

Technical Mathematics 1 was taught to three pilot classes (a total of 73 students) during the fall semester of 1965. The students were selected on the basis of a diagnostic test which had been specifically written for and used in the Technical Mathematics course during the five previous years. A representative cross-section of students was selected from the electrical, mechanical, and civil technologies since these technologies include about 65% of the entering students. The remaining 483 students were taught in conventional classes.

Topic-Unit Tests.

The course was divided into 11 topic-units. Parallel pre-tests and post-tests were given for each unit. The names of the topic-units and the means and medians for the pre-tests and post-tests are given in the table on the next page. (Note: The mean and median reported for post-tests in this table and subsequent tables are computed on the basis of original scores on the post-test. Even when retests are given, the retest scores are not used when computing the mean and median. The reported means and medians would obviously be higher if retest scores were used when computing them.)

TOPIC-UNIT TEST SCORES - TECHNICAL MATHEMATICS 1
PILOT CLASSES (1965)

TOPIC-UNIT	PRE-TEST MEAN	PRE-TEST MEDIAN	POST-TEST MEAN	POST-TEST MEDIAN
1. Operations With Signed Numbers	81%	84%	96%	98%
2. Estimation and Powers of Ten	55%	58%	88%	93%
3. Slide Rule Operations	24%	20%	82%	83%
4. Basic Algebraic Operations, and Solving Simple Equations	69%	74%	87%	91%
5. Geometric Formula Evaluation	67%	65%	85%	88%
6. Technical Measurement	37%	35%	81%	83%
7. Graphing	47%	44%	86%	88%
8. Formula Rearrangement	61%	60%	90%	92%
9. Systems of Equations	32%	12%	84%	94%
10. Quadratic Equations	40%	38%	85%	88%
11. Exponentials and Logarithms	25%	19%	73%	76%

Comparison With Conventional Classes.

We can determine the relative success of the pilot and conventional classes by comparing their dropout rates and by comparing their scores on a common final exam. Both of these comparisons are given in this section.

Dropout rate. There was a 19% dropout rate (14 of 73) in the pilot classes and a 39% dropout rate (188 of 483) in the conventional classes. Therefore, the dropout rate in the pilot classes was approximately one-half the dropout rate in the conventional classes. Presumably, the higher dropout rate in the conventional classes represented the loss of many lower ability and/or less motivated students.

Common Final Exam. A common final exam was administered to all students in all sections. The exam was written jointly by one member of the project staff and one conventional teacher. The items, which were restricted to topics which had been covered by both groups, were designed to test fundamental skills. Transfer items were avoided. A copy of the common final exam is given in Appendix F-1. The distribution of scores and the item analysis for the exam are included in Appendix F-2.

The maximum possible raw score was 85. The scores for the 59 pilot students ranged from 31 to 85; the scores for the 295 conventional students ranged from 2 to 84. The mean and median for the pilot students were 75% and 80% respectively; the mean and median for the conventional students were 57% and 61% respectively. The table on the next page gives a rough indication of the distribution of scores for each group.

COMPARISON OF FINAL EXAM SCORES
FOR PILOT CLASSES AND CONVENTIONAL CLASSES (1965)

Percent of Test Items Correct	Pilot	Conventional
90% or more	24%	5%
80% or more	51%	18%
70% or more	63%	34%
50% or more	93%	64%
30% or more	100%	83%

The percent of pilot students who received a score of 70% or better was approximately double the percent of conventional students who did so. It is interesting to note that 22% of the conventional students achieved a score which was lower than the lowest score of any student in the pilot groups. The superiority of the pilot classes was obtained in spite of the fact that the dropout rate in the pilot classes was considerably lower. There would undoubtedly have been a more marked difference in the distributions if the dropout rates in the two groups had been comparable.

Course Grades.

Grades were assigned to the pilot students on a strict percentage basis, with 70% required for a "D". Each student's grade was determined by weighting his post-test average 2/3 and his final exam score 1/3. The distribution of final grades for the pilot students was:

DISTRIBUTION OF COURSE GRADES
PILOT CLASSES (1965)

		Number of Students	Percent of Students
A	93% - 100%	16	27%
B	85% - 93%	17	29%
C	77% - 85%	17	29%
D	70% - 77%	9	15%
U	Below 70%	0	0%
		59	100%

The fact that 56% of the pilot students received either an "A" or "B" and no one failed was very encouraging.

Comment About the Pilot Classes.

This one semester with the pilot students was the only time during the history of the project when a direct comparison with a control group was possible. The scores of the pilot students on a common final exam were significantly higher than the scores of the students in the conventioanl classes, in spite of the fact that the dropout rate for the pilot students was only half the dropout rate of the conventional students. Therefore, the project staff was satisfied **that** the rough system of instruction which had been developed was a step in the right direction.

Technical Mathematics 1 (1966, 1967, 1968)

In this section, we will summarize the results obtained in Technical Mathematics 1 during the past three years. The topic-unit scores, final exam scores, dropout rates, and course grades for each of the three years will be given. One point that should be kept in mind when examining the data is the fact that a lengthy teachers' strike occurred during the 1968-69 school year. This strike, which began on January 1, 1969, included the last two classweeks and the final exam period for Technical Mathematics 1. Though the Learning Center was operated during this time by some teachers who did not participate in the strike, the strike had a tremendous effect on the morale and attendance of the students. Therefore, the results during that year are clearly lower than they would have been if the strike had not occurred.

Topic-Unit Tests.

The means and **medians** for the post-tests of the topic-units are given in the tables below and on the next page. When examining these tables, the following points should be considered:

(1) Pre-tests were abandoned after the 1966 school year.

(2) There has been some change in the topic-units from year to year. The major change has been a gradual expansion of the materials covering basic algebra and slide rule operations.

(3) The tests for topic-units have been changed, and therefore even the test scores for units with the same name are not directly comparable.

(4) The average post-test score for all students during this semester is generally over 90%.

TOPIC-UNIT TEST SCORES FOR TECHNICAL MATHEMATICS 1 (1966)

	PRE-TEST		POST-TEST	
TOPIC-UNIT	MEAN	MEDIAN	MEAN	MEDIAN
1. Number System and Signed Numbers	80%	71%	97%	100%
2. Non-Fractional Equations	73%	85%	93%	95%
3. Fractions and Fractional Equations	48%	55%	90%	95%
4. Formula Rearrangement	47%	50%	91%	95%
5. Powers of Ten	59%	63%	97%	98%
6. Estimation	53%	50%	93%	95%
7. Slide Rule	20%	0%	80%	84%
8. Logarithms	21%	10%	88%	94%
9. Technical Measurement	69%	72%	93%	94%
10. Graphing	19%	14%	84%	88%
11. Triangles and Trigonometry	62%	64%	88%	93%

TOPIC-UNIT TEST SCORES
FOR TECHNICAL MATHEMATICS 1 (1967)

TOPIC-UNIT	MEAN	MEDIAN
1. Number Line and Signed Numbers	96%	97%
2. Non-Fractional Equations I	96%	97%
3. Non-Fractional Equations II	92%	96%
4. Numerical and Literal Fractions	88%	92%
5. Fractional Roots and Fractional Equations	91%	95%
6. Formula Rearrangement	90%	96%
7. Introduction to Slide Rule	86%	88%
8. Estimation and Slide - First Test	77%	80%
Rule Operations - Second Test	85%	90%
9. Powers of Ten and Slide Rule	90%	92%
10. Slide-Rule - Powers and Roots	88%	92%
11. Reading and Constructing Graphs	94%	92%
12. Straight Line and Slope	84%	88%
13. Introduction to Logarithms	88%	92%
14. Systems of Equations	82%	85%
15. Triangles and Trigonometry	89%	93%

TOPIC-UNIT TEST SCORES
FOR TECHNICAL MATHEMATICS 1 (1968)

TOPIC-UNIT	MEAN	MEDIAN
1. Number Line and Signed Numbers	97%	98%
2. Non-Fractional Equations I	96%	97%
3. Non-Fractional Equations II	95%	96%
4. Multiplication and Division of Fractions	92%	94%
5. Addition and Subtraction of Fractions	88%	93%
6. Fractional Roots and Fractional Equations	90%	95%
7. Introduction to Graphing	95%	98%
8. Literal Fractions	89%	93%
9. Formula Rearrangement	91%	96%
10. Number System and Number Sense	95%	96%
11. Powers of Ten	94%	96%
12. Rounding and Rough Estimation	94%	97%
13. Introduction to Slide Rule	86%	88%
14. Slide Rule Multiplication and Division	82%	85%
15. Slide Rule Powers and Roots	88%	92%
16. Introduction to Logarithms	86%	89%
17. Triangles and Trigonometry	86%	90%

Final Exam Scores, Dropout Rates, Course Grades.

Though there have been revisions and changes in the final exams during the three years, the items on these final exams are designed to test fundamental skills. The mean and median scores for the final exams were 82% and 85% in 1966, 85% and 88% in 1967, and 83% and 87% in 1968. A copy of the final exam administered during the 1968-69 school year is given in Appendix G-1. The distribution of scores and the item analysis for that exam are included in Appendix G-2.

The dropout rates for both 1966 and 1967 were 21%. The official dropout rate for 1968 was 30%, but this figure includes 11% who completed all of the materials up to the time when the teachers' strike began. Therefore, we feel that the dropout rate would have remained constant if the strike had not occurred. A 20% dropout rate compares favorably to the 40% dropout rate which had occurred when the course was taught by conventional methods. The majority of the dropouts occur for nonacademic reasons. For example, in the 1966-67 school year, only 11 of 108 dropouts were actually failing the course at the time of their withdrawal.

In each year, course grades for the students were determined on a strict percentage basis, with the average on the post-tests weighted 2/3 and the score on the final exam weighted 1/3. The distribution of grades for each year is given in the table below:

DISTRIBUTION OF COURSE GRADES
TECHNICAL MATHEMATICS 1 (1966, 1967, 1968)

		1966		1967		1968	
		N	%	N	%	N	%
A	93% - 100%	141	36%	125	33%	148	38%
B	85% - 93%	136	35%	148	39%	103	26%
C	77% - 85%	65	16%	61	16%	55	14%
D	70% - 77%	40	10%	23	6%	23	6%
U	Below 70%	11	3%	20	6%	7	2%
I	Incomplete	0	0%	0	0%	53	14%

In 1966, 71% received either an "A" or "B"; in 1967, 72% received either an "A" or "B". In 1968, only 64% received either an "A" or "B", but this figure is certainly low because 14% of the students received an "incomplete" as a result of the teachers' strike.

Comments About Technical Mathematics 1 (1966, 1967, 1968).

In spite of the changes in the classroom procedure, the commonality of the results from year to year was rather striking. For example:

(1) The post-test average for topic-units was usually around 90%.

(2) The mean and median on the final exams were in the mid-80's.

(3) The dropout rate was approximately 20%.

(4) The percent receiving "A's" or "B's" was approximately 70%.

Though there were some deviations from this pattern because of the teachers' strike in 1968, we would expect similar results in future years. Improvements in the system of instruction should compensate for any "Hawthorne effect" which might have been operative in the early years of the project.

Technical Mathematics 2 (1967, 1968, 1969)

In this section, we will summarize the results obtained in Technical Mathematics 2 during the past three years. Topic-unit scores, final exam scores, dropout rates, and course grades for each of the three years will be given. The teachers' strike also had an effect on the results during 1969. The strike was not concluded until after the first three weeks of the second semester. Though the Learning Center was operated, many students manifested a nonchalance during that semester which had not been present in prior years.

Topic-Unit Tests.

The means and medians for the post-tests of the topic-units are given in the tables below and on the next page. When examining these tables, the following points should be noted:

(1) No pre-tests were ever given during this semester. If they had been given, very low scores would have been expected.

(2) There has been some change in the topic-units and the order in which they were taught from year to year.

(3) The tests for the topic-units have been revised and changed, and so the test scores for units with the same name are not directly comparable.

(4) The average post-test score for all students during this semester was slightly over 85%.

TOPIC-UNIT TEST SCORES
FOR TECHNICAL MATHEMATICS 2 (1967)

TOPIC-UNIT	MEAN	MEDIAN
1. Oblique Triangles I	94%	94%
2. Vectors	85%	87%
3. Systems of Equations	89%	95%
4. Oblique Triangles II	85%	89%
5. Quadratic and Radical Equations	87%	92%
6. Geometry and Applied Trigonometry	84%	87%
7. Logarithms: Laws and Formulas	82%	88%
8. Exponentials: Base "e" and Natural Logarithms	83%	83%
9. Sine Waves	90%	92%
10. Further Topics in Trigonometry	90%	93%

TOPIC-UNIT TEST SCORES FOR TECHNICAL MATHEMATICS 2 (1968)

TOPIC-UNIT	MEAN	MEDIAN
1. Vectors	88%	90%
2. Technical Measurement	92%	93%
3. Quadratic Equations	88%	91%
4. Radicals and Radical Equations	81%	86%
5. Oblique Triangles I	96%	94%
6. Oblique Triangles II	88%	90%
7. Sine Waves	90%	94%
8. Further Trig Topics	90%	92%
9. Complex Numbers	92%	94%
10. Geometry and Applied Trigonometry - Part I	78%	82%
- Part II	84%	83%
11. Logarithms: Laws and Formulas	85%	88%
12. Exponentials: "Base "e" and Natural Logarithms	83%	83%

TOPIC-UNIT TEST SCORES FOR TECHNICAL MATHEMATICS 2 (1969)

TOPIC-UNIT	MEAN	MEDIAN
1. Vectors	85%	87%
2. Trig Ratios of General Angles	93%	94%
3. Complex Numbers	89%	92%
4. Radicals and Radical Equations	82%	87%
5. Systems of Equations	83%	86%
6. Oblique Triangles	89%	91%
7. Sine-Wave Analysis	83%	86%
8. Straight Line and Slope	86%	90%
9. Quadratic Equations	88%	91%
10. Geometry and Applied Trigonometry - Part I	72%	73%
- Part II	82%	83%
11. Common and Natural Logarithms	88%	90%
12. Further Trig Topics	87%	89%
13. Logarithms: Laws and Formulas	78%	83%

Final Exam Scores, Dropout Rate, Course Grades.

Though there have been revisions and changes in the final exams during the three years, the items on these final exams are designed to test fundamental skills. The mean and median scores for the final exams were 77% and 78% in 1967, 82% and 85% in 1968, and 81% and 83% in 1969. A copy of the final exam administered during the 1968-69 school year is given in Appendix H-1. An item analysis and a distribution of scores for that exam are included in Appendix H-2.

The dropout rates during the three years were 12% (1967), 13% (1968), and 16% (1969). Hopefully, the slight increase in the dropout rate in 1969 is related to the effect of the teachers' strike on the students' morale.

Course grades were again determined on a strict percentage basis. In the basic formula for determining grades, the post-test average was weighted 2/3 and the final exam score 1/3. In 1967, a midsemester exam score was included in the post-test average; it was given the weight of two post-tests. In 1969, five comprehensive exam scores were included in the post-test average; each comprehensive exam score was given the weight of one post-test. The distribution of grades for each year is given in the table below:

DISTRIBUTION OF COURSE GRADES
TECHNICAL MATHEMATICS 2 (1967, 1968, 1969)

		1967		1968		1969	
		N	%	N	%	N	%
A	93% - 100%	41	15%	46	19%	36	16%
B	85% - 93%	97	37%	107	44%	83	38%
C	77% - 85%	74	28%	57	23%	44	20%
D	70% - 77%	41	15%	22	9%	44	20%
U	Below 70%	13	5%	11	5%	14	6%

The percent of students receiving either an "A" or "B" was 52% in 1967, 63% in 1968, and 54% in 1969. In 1969, the scores on the five comprehensive tests tended to lower the students' averages, and therefore, the percent receiving an "A" or "B" was somewhat down and the percent receiving a "D" was somewhat up.

Comments About Technical Mathematics 2 (1967, 1968, 1969).

The results in Technical Mathematics 2 are generally lower than those in Technical Mathematics 1 because the topics covered are more difficult. And though there is certainly some commonality in the results for the three years, the commonality is not as striking as it was for Technical Mathematics 1. In general, the following statements can be made:

(1) The post-test average for topic-units was usually in the mid or high 80's.

(2) The mean and median on the final exams were in the high 70's or low 80's.

(3) The dropout rate ranged from 12% to 16%.

(4) The percent receiving "A's" or "B's" ranged from 52% to 63%.

Hopefully, the results in 1970 will be somewhat improved over those in 1969 when the teachers' strike had an effect on morale. The project staff is confident that improvements in the system of instruction will overcome any "Hawthorne effect" advantages which might have been operative in the early years of the project.

Comprehensive Exams in Technical Mathematics 2 (1969)

Five comprehensive exams were given during Technical Mathematics 2 in 1969. These five exams reviewed the topics covered in both semesters. They covered the following content areas: arithmetic, basic algebra, advanced algebra, graphing, and trigonometry. The students were told that the scores on these tests would count toward their final grades. The exams were given in an attempt to force the students to review and integrate topics which had been taught and tested separately. The staff felt that this review would serve as distributed practice and increase the probability of long-range retention of learning.

Each comprehensive exam was designed so that it could serve as a measure of retention for topics covered in both courses. Each item in each exam was parallel to an item which had appeared in some topic-unit test in one of the two semesters. Therefore, it was possible to specify the amount of retention on each item by comparing its difficulty level on the comprehensive exam with the difficulty level of the parallel item on a topic-unit test.

Though the students were notified in advance about the specific topics which would be included in each exam, there were no review sessions before the exams. Any reviewing which the students did was self-motivated and self-controlled; no measure of the amount of actual reviewing is available. Since the comprehensive exams were interspersed among assignments covering new topics, the students did not have an unlimited amount of time to devote to review. Formal review sessions were avoided in an effort to get as pure a measure of retention as possible within the context of a real exam in a real course.

In the analyses of these exams, only those students were included for whom a complete set of data was available. A complete set of data included not only the comprehensive exam itself, but all of the prior topic-unit tests on which the parallel items appeared. Since items from topic-unit tests in Technical Mathematics 1 were included in several comprehensive exams, the new students in Technical Mathematics 2 who had not taken Technical Mathematics 1 during the previous semester were necessarily excluded from those analyses. Some students were also excluded because one or another of their tests had been misplaced.

After each comprehensive exam, each student was required to rework all of his incorrect items. Students who did not achieve a minimum acceptable score were also required to take a retest. For students in the "regular" class, reworking the incorrect items constituted the "tutoring" for the retest. This minimum amount of tutoring was tried because the staff was looking for a very quick and efficient method of accomplishing the necessary recall. For students in the "slow" class, merely reworking incorrect items was not a sufficient method of tutoring. The teachers spent as much individual time with the "slow" students as possible. The teachers did not hold to a strict "minimum acceptable score" criterion for the retesting. For example, a student was not retested if many of his errors on the exam could be traced to a misunderstanding of a single principle.

Arithmetic.

The arithmetic comprehensive exam given in April, 1969, was a parallel form of the 50-item Pre-Test: Arithmetic which had been given in September, 1968, at the beginning of Technical Mathematics 1. Parallel forms of 25 of the 50 items had also appeared in various post-tests during Technical Mathematics 1.

A copy of one form of this exam is given in Appendix B-1. A distribution of scores and an item analysis are given in Appendix B-3. A complete set of data was available for 204 students. The mean and median on the pre-test in September, 1968, for these 204 students were 68% and 72% respectively; their mean and median on the comprehensive exam were 90% and 92% respectively. The gains for each of the six sub-sections of the test are given in the table below:

MEAN SCORES FOR SUB-SECTIONS OF ARITHMETIC COMPREHENSIVE EXAM

		Pre-Test (Sept., 1968)	Comp. Exam (April, 1969)	Gain
Whole Numbers	(4 items)	84%	94%	+10%
Decimals	(4 items)	81%	92%	+11%
Percents	(6 items)	78%	91%	+13%
Number System	(10 items)	62%	85%	+23%
Number Sense	(4 items)	80%	93%	+13%
Fractions	(22 items)	61%	90%	+29%

There were gains in each sub-section of the test, even in the first three which were not specifically covered during the course.

Comparing the difficulty levels of 25 items on the arithmetic comprehensive exam with the difficulty levels of the 25 parallel items in topic-unit tests, the students performed better on 7, worse on 17, and the same on 1. For the complete set of 25 items, the average loss per item on the comprehensive exam was 4%.

No formal retesting was done after this exam because the students typically performed at a high level.

Basic Algebra.

This exam was a parallel form of the 30-item Pre-Test: Algebra which was administered at the beginning of Technical Mathematics 1 in September, 1968. It was readministered without warning in February. Tutoring was done at that time with students who had performed poorly. Parallel forms of the same test were then administered in April, 1969, as a comprehensive exam. Therefore, parallel forms of the same test were administered at three times during the year. Furthermore, parallel forms of 22 of the 30 items had appeared in post-tests during Technical Mathematics 1.

A copy of one form of this exam is given in Appendix C-1. A distribution of scores and an item analysis are given in Appendix C-3. A complete set of data was available for 196 students. Their mean and median on the pre-test in September were 44% and 40% respectively. Their mean and median on the unannounced test in February were 83% and 87%, respectively. Their mean and median on the comprehensive exam in April were 91% and 93%, respectively. The means for each of the six sub-sections of the test during each of the three administrations is given in the table below:

MEAN SCORES FOR SUB-SECTIONS OF BASIC ALGEBRA COMPREHENSIVE EXAM

		Pre-Test Sept., 1968	Retest Feb., 1969	Comp. Exam April, 1969
Signed Numbers	(6 items)	60%	91%	94%
Powers of Ten	(2 items)	26%	86%	87%
Algebraic Fractions	(2 items)	40%	74%	86%
Non-Fractional Equations	(6 items)	62%	92%	96%
Fractional Equations	(6 items)	24%	69%	82%
Formula Rearrangement	(8 items)	34%	81%	92%

Comparing the difficulty levels of the 22 items on the comprehensive exam with the difficulty levels of the 22 parallel items on topic-unit tests, the students performed better on 4, worse on 14, and the same on 4. For the complete set of 22 items, the average loss per item on the comprehensive exam was 5%.

No formal retesting was done after this exam because the students typically performed at a high level.

Advanced Algebra.

This 21-item comprehensive exam was given in May, 1969. It covered these topics: (1) radical equations and formulas, (2) quadratic equations, and (3) systems of equations and formulas. All of these topics had been taught during Technical Mathematics 2, and all of the items were parallel to items which had appeared in topic-unit tests during the second semester. A copy of one form of this exam is given in Appendix I-1. A distribution of scores and an item analysis are given in Appendix I-2. Using the item difficulty levels from the various topic-unit tests, the predicted mean for the exam was 80%. The obtained mean for 216 students was 71% (the median was 76%). The predicted and obtained means for the various sub-sections of the exam were:

MEAN SCORES FOR SUB-SECTIONS OF ADVANCED ALGEBRA COMPREHENSIVE EXAM

		Predicted Mean	Obtained Mean
Equations Involving Radicals and Squares	(4 items)	77%	66%
Quadratic Equations	(6 items)	86%	65%
Systems of Two Equations and Formulas	(7 items)	80%	80%
Systems of Three Equations and Formulas	(4 items)	72%	72%

Though the overall mean reflected some forgetting, this forgetting occurred with radical equations and formulas and quadratic equations, but not with systems of equations.

A total of 95 students (67 from the regular class and 28 from the slow class) were retested 12 days after the original administration of the exam. All 95 students had achieved a raw score of 15 or less (71% or less). (Note: 7 students who obtained scores below this cut-off point were not retested for one reason or another.) The tutoring for the "regular" students was limited to reworking incorrect items. There was no standard procedure for tutoring the "slow" students; they were given as much personal attention as possible by a teacher who had many other responsibilities. The mean and median on the original test and retest for the various groups are given in the table below:

MEAN AND MEDIAN SCORES
FOR RETEST OF ADVANCED ALGEBRA COMPREHENSIVE EXAM

		Original Test		Retest	
		Mean	Median	Mean	Median
Regular Students	(n = 67)	55%	57%	80%	81%
Slow Students	(n = 28)	41%	43%	63%	62%
Overall	(n = 95)	51%	52%	75%	76%

Though some students still had low scores on the retest, there were, in general, substantial gains. In fact, if the retest scores for these 95 students are substituted for their original scores in the overall distribution for 216 students, the mean and median for the distribution shift to 81% and 86% respectively. This mean of 81% is higher than the predicted mean of 80%.

Graphing.

This 37-item comprehensive exam was given in May, 1969. It included the following topics: (1) graphing simple equations and formulas, (2) the straight line; intercepts and slope, (3) sine wave graphs, and (4) exponential graphs. All topics, except the first one, were taught during Technical Mathematics 2. All of the items were parallel to items which had appeared in post-tests during one of the two semesters. A copy of one form of this exam is given in Appendix J-1. A distribution of scores and an item analysis are given in Appendix J-2. Using the item difficulty levels from the various post-tests, the predicted mean for the exam was 88%. The obtained mean for 197 students was 79% (the median was 81%). The predicted and obtained means for the various sub-sections of the exam were:

MEAN SCORES FOR SUB-SECTIONS OF GRAPHING COMPREHENSIVE EXAM

		Predicted Mean	Obtained Mean
Graphing Simple Equations and Formulas	(11 items)	96%	92%
Straight Line: Intercepts and Slope	(14 items)	84%	70%
Sine-Wave Graphs	(9 items)	83%	75%
Exponential Graphs	(3 items)	97%	87%

There was some forgetting in each sub-section of the exam.

A total of 48 students (24 from the regular class and 24 from the slow class) were retested five days after the original administration of the exam. All 48 students had achieved a raw score of 26 or less (70% or less). (Note: 8 students who obtained scores below this cut-off point were not retested for one reason or another.) The tutoring for the "regular" students was again limited to reworking incorrect items. There was again no standard procedure for tutoring the "slow" students; the teachers claimed that enough time to do a thorough job was not available. The mean and median on the original test and retest for the various groups are given in the table below:

MEAN AND MEDIAN SCORES
FOR RETEST OF GRAPHING COMPREHENSIVE EXAM

		Original Test		Retest	
		Mean	Median	Mean	Median
Regular Students	(n = 24)	60%	62%	87%	88%
Slow Students	(n = 24)	57%	59%	73%	74%
Overall	(n = 48)	58%	59%	80%	81%

Though there were gains for both groups, the gains for the "regular" students were more substantial. If the retest scores for these 48 students are substituted for their original scores in the overall distribution for 197 students, the mean and median for the distribution shift to 84% and 86%, respectively. This new mean is still lower than the predicted mean of 88%, but the new mean includes scores for 149 students who were not retested.

Trigonometry.

This 38-item comprehensive exam was also given in May, 1969. It included the following topics: (1) right triangles, (2) general angles, (3) arcsin notation, radians, and identities, (4) vectors, (5) one applied problem, and (6) either complex numbers for the electrical students or oblique triangles for the other students. All of these topics, except right triangles, had been taught during Technical Mathematics 2. Each item on the exam was parallel to an item which had appeared in a topic-unit test during one of the two semesters. A copy of this exam is given in Appendix K-1.

A distribution of scores and an item analysis are given in Appendix K-2. Using the item difficulty levels of these parallel items, the predicted mean for the exam was 87%. The obtained mean for 185 students was 75% (the median was 76%). The predicted and obtained means for the various sub-sections of the exam were:

MEAN SCORES FOR SUB-SECTIONS OF TRIGONOMETRY COMPREHENSIVE EXAM

		Predicted Mean	Obtained Mean
Right Triangles	(8 items)	94%	93%
General Angles	(8 items)	90%	76%
Arcsin notation, radians, identities	(6 items)	87%	84%
Vectors	(7 items)	81%	64%
Applied Problem	(1 item)	65%	66%
Complex Numbers	(8 items)	83%	63%
Oblique Triangles	(8 items)	87%	61%

Though there was some forgetting in each sub-section (except the "applied problem" section), the amount of forgetting in some sub-sections is clearly greater.

There was no retesting after the trigonometry exam because the semester was almost over. The teachers felt that tutoring for a retest and administering it would have been too hectic, if not impossible.

Comments About Comprehensive Exams.

In general, the staff felt that the effort devoted to the comprehensive exams was very worthwhile. Though the amount of time given to reviewing by the students was unknown, each student was, at least, subjected to an assessment of the major-terminal objectives of both courses. The general complaint, however, was that the period during the course when these exams were given was quite hectic. While the students were taking these exams, they were also responsible for learning new material. And generally, the students who had difficulty learning the new material also did poorly on the comprehensive exams. Therefore, the amount of time they had to spend being tutored and retested on both topic-unit tests and on comprehensive exams got to be overwhelming.

The level of retention was generally good. Ordinarily, the obtained mean was approximately 10 percentage points lower than the predicted mean. The tutoring and retesting of students with low scores suggested these two conclusions:

(1) If a student had previously mastered a topic, his relearning could be accomplished quite easily even if his recall was deficient.

The evidence for the ease of relearning were the "regular" students whose retest scores rose dramatically after merely reworking the problems they had wrong. Of course, since the retest included parallel items, it is impossible to tell whether this relearning was related merely to these specific items or whether it had a more general effect.

(2) If a student had not previously mastered a topic, his relearning could not be accomplished easily.

The teachers' experience with some of the "slow" students suggested that there was no easy way to polish skills which these students never really had. In fact, it seemed that it was not a question of relearning but one of original learning.

Obviously, some type of formal review before these exams would have raised the exam scores. However, with long-range retention as the major goal of the comprehensive exams, some experiments would have to be conducted to determine which of various review procedures is better. One procedure is to give a formal review before the exam. Another procedure is to give the exam without a formal review and then tutor afterwards. Other procedures which incorporate both formal reviews and tutoring are possible.

Student Attitude Questionnaires

A questionnaire concerning attitudes towards the system of instruction was filled out by most students at the end of Technical Mathematics 2 in both 1967 and 1969. The 244 students who filled out the questionnaire in 1967 had been taught in satellite classrooms. The 214 students who filled out the questionnaire in 1969 had been taught completely in the Learning Center. Though the questionnaire itself was much longer, only the students' responses to the more significant items are listed below:

(1) "How much did you learn in this course compared to previous math courses?"

	1967		1969	
	Number of Students	Percent of Students	Number of Students	Percent of Students
Much More	127	52%	124	58%
More	85	35%	71	33%
Same Amount	14	6%	6	3%
Less	11	4%	10	5%
Much Less	4	2%	3	1%
No Response	3	1%	---	---
	244		214	

(2) "How much did you like this course compared to previous math courses?"

	1967		1969	
	Number of Students	Percent of Students	Number of Students	Percent of Students
Much More	114	47%	121	57%
More	86	35%	70	33%
Same Amount	25	10%	19	9%
Less	15	6%	3	1%
Much Less	4	2%	1	0%
	244		214	

(3) "How hard did you work in this course compared to previous math courses?"

	1967		1969	
	Number of Students	Percent of Students	Number of Students	Percent of Students
Much More	36	15%	42	20%
More	75	31%	95	44%
Same Amount	53	21%	38	18%
Less	61	25%	29	13%
Much Less	19	8%	10	5%
	244		214	

(4) "If you took another math course, would you prefer to study math with this same type of system?"

	1967		1969	
	Number of Students	Percent of Students	Number of Students	Percent of Students
Yes	218	89%	207	97%
No	21	9%	7	3%
No Response	5	2%	---	---
	244		214	

(5) "Would you prefer to have this type of system in other courses besides math?"

	1967		1969	
	Number of Students	Percent of Students	Number of Students	Percent of Students
Yes	192	79%	158	74%
No	43	18%	56	26%
No Response	9	3%	---	---
	244		214	

In general, the student reaction to the system of instruction was very positive. The staff was pleased with the fact that the students seemed even more positive in 1969 than in 1967. The percent who claimed they "learned more" increased from 87% in 1967 to 91% in 1969. The percent who claimed they "like the course more" increased from 82% in 1967 to 90% in 1969. The percent who claimed they would "prefer this system of instruction in further math courses" increased from 89% in 1967 to 97% in 1969. The increase in positive attitudes over a two-year period suggests that the students' enthusiasm for a system of instruction of this type will continue.

The spontaneous comments of the students on the questionnaires were also interesting. Many were impressed with the teachers' interest in individual students. Many commented that they had learned more math in one year than they had learned in all of their previous math courses. Some suggested quite emphatically that math should be taught with a comparable system in high schools. Many were quite emphatic when suggesting other courses in which this same type of instruction could be used.

General Discussion of Results at Milwaukee Area Technical College

Novel System of Instruction.

The system of instruction which has been developed over a four-year period is novel compared to the traditional method of teaching mathematics. The major components of the system are programmed materials, daily assessment, and tutoring when it is required. Based on the premise that learning occurs to the extent that an instructional system gains control over the motivation and learning process of each student, the system has been designed to offer daily personal attention to each student in spite of the fact that a very large number of students are serviced. The management of all aspects of the system from a Learning Center has given the system a type of flexibility which is virtually impossible under any other type of management. This flexibility has made possible such services as special treatments for fast, regular, and slow learners, tutoring at any time during the day, and an efficient method for handling the make-up work of absentees. Besides the increased flexibility which it provides, the use of a Learning Center is more economical. Its economy has been increased by the use of teacher aides and clerical help.

Success of the System.

Though the system of instruction is neither completed not perfect, the three specific goals of the project for the math course have been accomplished. That is:

(1) The course content has been revised to make it more relevant to the needs of industrial technicians. (The content is really relevant to the needs of any student who intends to study elementary science.)

(2) The dropout rate has been significantly reduced.

(3) The level of achievement has been substantially increased.

Unfortunately, the improvement in the math instruction has had little effect on the overall dropout rate (about 65%) in the technical curricula. In retrospect, this hope for a global effect was somewhat naive. The math course comprises only slightly over 10% of the course credits in the technical curricula, and while the method of math instruction has changed, the methods in the other courses have remained very traditional. The hope for a reduction in the overall dropout rate was based on an inflated opinion of the role of mathematics in the success of other courses in the technical curricula. Math skills have little relevance in the general education courses included in the technical curricula. Though math skills are a necessary condition for success in the science courses and in many of the technical courses, they are certainly not a sufficient condition for this success. Granted sufficient math skills, instruction in science and technical courses still depends on the teachers' ability to communicate concepts and principles. Whether these concepts and principles can be communicated to the majority of the technical students by traditional methods is highly questionable.

Aside from instructional methods, there are also some general factors which contribute to the overall dropout rate. The open-door policy of the school encourages the enrollment of students from the bottom quarter of high school classes, and many of these students simply do not have the ability to succeed in the program. Furthermore, because of the lack of vocational-technical programs in the high schools in the Milwaukee area and elsewhere in Wisconsin, students enroll who are poorly informed about the nature of technical training and technical jobs. Many of these students drop out from lack of interest after they become better acquainted with the technical area. Others drop out because the curricula are so designed that the first year contains a heavy dose of general education courses, and they lost interest when subjected to courses which they do not perceive as being relevant to their interests or needs. And of course, there are always a certain number who are drafted into the armed services, or who have to take a job for personal reasons.

Reason for the Success of the System.

It is impossible to determine the relative contribution of each aspect of the instructional system to its overall success. One clear reason for the success is that the content of the course takes into account the entry skills and learning capabilities of the students. Another clear reason is the amount of teacher control over student behavior which the system permits. It is difficult to assess the relative contribution of the programmed materials, the daily testing, and the tutoring. The programmed materials are more or less self-sufficient depending upon the type of learner. They seem to be self-sufficient with the fast learners since these students can and do learn without attendance at the regular classes. They are not self-sufficient with the students in the regular classes, although the required tutoring in those classes can be accomplished during class time even though a large number of students are involved. The materials are clearly not self-sufficient for the slow students who need a great deal of personal attention from the teachers. Fortunately, the number of students in this category is relatively small. We feel that the daily tests have an effect for all students, since they give the students an objective assessment of their own progress and they give the teachers a method of controlling each student's effort and progress. In fact, it seems that daily tests would be beneficial with any method of instruction.

Student Reaction.

The attitude of the students towards the system of instruction has been very positive. This positive attitude is undoubtedly related to the level of achievement which they attain. Though many of the students begin the course with a fair amount of anxiety because of their past experience with math courses, their success dissipates this anxiety and replaces it with a confidence in their ability to learn. The system makes a deliberate attempt to raise the level of aspiration of the students, and it seems to be successful in doing so. Many of the teachers claim that, under conventional teaching, they had seriously underestimated the math learning potential of the technical students.

Though a lack of student motivation was anticipated at the beginning of the project, a motivation problem did not materialize. Except for a small number of students, the student motivation has been more than satisfactory. Their motivation is controlled by a method of daily checks which have been deliberately included in the system of instruction. But even when a fair amount of pressure is exerted on the students, the students respond fairly well. In fact, as the teachers have become more adept at controlling the students' behavior, the number of emotional encounters between teacher and student has decreased to a minimum. It seems plausible that the motivation of the students is related to various factors such as the sensibleness of the system of instruction, the amount of personal attention which they receive, and their own success. Perhaps the motivation of students of this type has also been underestimated. It is quite possible that many of them are well-motivated when they begin courses, but that their motivation disappears when their efforts to learn prove to be unsuccessful.

Teacher Reaction.

Though asked to assume a new role, the cooperation of the teachers during the course of the project has been very good. Ten different teachers have been involved at some time during the past three years. They have worked out more or less well depending upon their skills at tutoring and controlling students plus their ability to function as a member of a highly organized group. The teachers who have remained with the project have developed skills in tutoring and controlling students which are highly related to the success of the project. In fact, their skills as a group have become so highly developed that it would be difficult for any new group to compete with them at this time.

Since a system of instruction cannot survive in our educational system if teacher reaction to it is negative, there has been a necessary concern about the attitudes of the teachers. At first, the teachers were apprehensive. Even though most of them were hired with the understanding that they would work with an experimental project, it was still difficult for them to have a system imposed upon them by the project staff. And in spite of their willingness to try something new, there was a conflict between their new role and the role which an ordinary teacher expects to play in his profession. Some teachers admitted that they missed lecturing and were disturbed by the fact that the students were learning without their lectures. Most were fairly conscious of their status, with a fear that their new role was merely that of a technician or bookkeeper.

As the learning system evolved and the teachers became more involved in it, their attitudes became more positive. There are various reasons why this change in attitude occurred. Some of the reasons are more general, and some are specifically related to the use of a Learning Center. The more general reasons are:

(1) The instructional system has been successful, and most teachers are interested in the achievement level of their students.

(2) It became clearer to them that their role was also a highly professional one. That is, they saw that the system was designed so that they could cope with the learning problems of individual students. Most of them find this type of activity rewarding.

(3) They became more involved in the decision-making process of the project, and so their ideas and suggestions had an influence on changes in the system.

The reasons related specifically to the use of a Learning Center are:

(1) The efficiency of the Learning Center made it possible to limit each teacher's contact with students to his regularly assigned periods. By the constant availability of manpower in the Center, a fair amount of teacher-student contact in the teachers' offices was eliminated.

(2) The increased use of para-professional personnel (teacher aides and clerical help) freed the teachers from many of their non-professional duties.

The fact that the teachers' attitudes have become more positive does not mean that all of them are wildly enthusiastic about the system of instruction. As one teacher put it, "The teachers who are assigned to the Learning Center do not object." This lack of objecting is clearly a gain since assignments to the Technical Mathematics course before the project began were often viewed with distaste and were accepted with reluctance. Many of the teachers still prefer a mixed schedule in which some of their classes are taught conventionally. Teaching a mixed schedule of this type has some advantage to the teachers since they usually have more time available to prepare their other classes.

Working in a highly organized learning system of this type for any period of time has a definite effect on the attitude of teachers towards math education in general. They become much more aware of the need to examine such things as the relevance of topics, the entry skills of the students, the amount of learning which occurs, and the type of assessment which is used. They no longer assume that students understand topics merely because they were covered in a previous course. As they begin to concentrate on student learning, they begin to see the absurdity of rushing through topics at a pace which is too fast for many of the students. As one teacher said, "A teacher who works in this system will never be the same as a conventional teacher."

Course Content.

Though the content and sequencing of the course as it now stands is "adequate" for most technologies, both could be improved. Math topics still come up in technical courses before they are covered in the math course, and topics which are needed in some technical courses are not taught. The major reason for the content and sequencing problems is the fact that most of the first semester is spent with topics which are remedial in nature. With the present entry skills of the students, this remedial work is absolutely essential. But necessary as the remedial work is, it does interfere with a content and sequencing which would be more acceptable.

Of the additional topics which could be added, some would benefit all students whereas others would only benefit students in specific technologies. Further work with calculation and numerical fluency would benefit all students. And there are a whole series of topics, which might be called the "quantitative aspects of science," that are really mathematical in nature and would be valuable for all students. The list below suggests what some of these topics are:

Basic Measurement Concepts
Systems of Measurement Units
Formula Evaluation
Empirical Graphing
Variation
Derivations

Besides the topics which would benefit all of the students, the students in electrical need a unit on sine-wave resultants, and the students in mechanical and civil need more units on plane geometry.

Though some resequencing could be done with the present content, no instructional time is left for additional topics. Until the entering students are better prepared or a pre-technical program is introduced for those with low entry skills, most of the topics listed above will have to be ignored. The only other alternative would be an increase in the number of courses and credits for mathematics within the context of the regular technical programs.

Problem Solving and Retention.

Meaningful verbal problems for technicians require a knowledge of the principles of science and technology. Since the staff realized that the students did not have a common core of such principles upon which verbal problems could be based, no specific units were devoted to verbal problem-solving. The development of problem-solving skills of this type has been left to the science and technology teachers. Hopefully, a similar system of instruction will be developed for the Technical Science (physics) course within the next few years. If this development is done, formal strategies for verbal-problem solving will be included in that course. If these strategies are taught properly, they should generalize to the other technical courses provided that the concepts and principles in those courses are adequately communicated.

The major effort of the project up to this time has been concentrated on original learning rather than on retention. This concentration on original learning seems sensible because retention is not a problem until learning occurs. Now that a reasonable level of learning has been attained, more thought and effort has to be given to the problem of long-range retention. The comprehensive exams which were given in the 1968-69 school year were a crude step in that direction. Some review materials which could be used in conjunction with the comprehensive exams are needed since it is cumbersome to use programmed materials for this review. Furthermore, since perfect long-range retention is probably an unreasonable expectation, some short review materials should be available for the students after the math course has ended.

Fast Learners.

In conjunction with the overall goals of the project, the emphasis in the system of instruction has always been placed on the average and below-average students. Because of this emphasis, the faster learner with high entry skills has been somewhat ignored. Though the special class for fast learners was initiated so that these students would not be subjected to the daily class routine, this treatment really avoids the problem more than it solves it. The problem, of course, occurs in Technical Mathematics 1. Though few students could pass the final exam in Technical Mathematics 1 at the beginning of the semester, perhaps as many as 25% could be given a much more abbreviated course. However, since materials for such a course are not available, an abbreviated course has not been possible. Hopefully, the development of some abbreviated materials will make such a course possible at some time in the future.

CHAPTER 4

EXPERIMENTAL CLASSES IN HIGH SCHOOLS

During the past two years, some of the materials developed for the Technical Mathematics course have been used on an experimental basis in various Milwaukee area high schools and in one junior high school. The schools which have participated in these field tests and the type of class in which the materials were used are listed below:

Pius XI High School	a Technical Mathematics course (juniors and seniors)
West Division High School	a substitute for general math (sophomores, juniors, seniors)
Franklin High School	a substitute for general math (sophomores)
Hamilton-Sussex High School	a class for emotionally disturbed freshmen
Academy of Basic Education	a replacement for the regular course for average-ability 7th, 8th, and 9th graders

These field tests have included students of various ages with various learning abilities and problems.

There is no question about the usefulness of the content of the Technical Mathematics materials in high schools. The content is useful because a majority of high school students cannot cope with college-preparatory math courses, and the general math courses which are offered as an alternative ordinarily have no well-conceived goal. The content is ideal for students who intend to become technicians, apprentices, or skilled tradesmen. It is also suitable for college-bound students who do not intend to become mathematicians, scientists, or engineers. Besides offering a content which is not found in available textbooks, the programmed materials offer an alternate method of instruction for students with whom the lecture-discussion method is not successful.

The project staff has been interested in these experimental high school classes for two major reasons. First of all, a Technical Mathematics sequence in local high schools would clearly raise the level of entry skills for the students who eventually enroll in technician training. Based on the results of the pre-tests in arithmetic and algebra, this level of entry behavior is quite low. If the level of entry behavior could be raised, less remedial work would be necessary in Technical Mathematics 1, and the content of the Technical Mathematics sequence could be upgraded. Better preparation in high school is a more efficient solution than the use of pre-technical mathematics courses which delay the student's entrance into the technical programs themselves. Second, the high school classes offer a direct method of recruiting for technician-training programs. With the growing need for technicians, some straightforward method for recruiting is absolutely essential. It is essential because many high school counselors are uninformed about technical careers or are somewhat negative towards any post-high-school training which is not purely academic.

The use of programmed materials is usually a new experience for high school teachers. When programmed materials and continual assessment are used, the teacher's attention is focused directly on the learning process. It soon becomes obvious that it is silly for a student to proceed if he has not mastered learning sets which are needed in the next segment of the instruction. The students inevitably proceed at different rates because of different ability-levels and different motivation-levels. Therefore, the teacher has to develop a classroom procedure which copes with the individual differences which are present. In terms of the development of a suitable classroom procedure, the field tests in the high schools have been more or less well-controlled. Since the amount of time which the project staff could devote to interactions with the high school teachers was often limited, the materials were sometimes used by teachers who had little more than a one-hour introduction to their use by some member of the project staff. In other instances, however, it was possible to devote a greater amount of time to orient high school teachers in the use of the materials.

The major attention of the project staff was devoted to the experimental classes at Pius XI High School and West Division High School. Therefore, only the data from these two schools will be reported. In many ways, this data is the most interesting, anyway, since a two-year Technical Mathematics sequence has been initiated at Pius XI High School, and West Division is a high school in the core area of Milwaukee.

Pius XI High School

Pius XI High School is a large, private (Catholic) coeducational school on the outskirts of Milwaukee. Like most private schools, its student body is somewhat above average in ability. The Milwaukee Area Technical College's relationship with this school is probably as good or better than its relationship with any other secondary school in the Milwaukee area. In 1963 and 1964, the counselors and teachers at Pius XI High School had discussed the possible introduction of a high school Technical Mathematics course with the teachers at the Milwaukee Area Technical College. The course was never begun, however, since suitable materials were not available. Therefore, though all students were required to take at least two years of college-preparatory mathematics, no mathematics course was offered to juniors and seniors who were not approved for enrollment in further college-preparatory mathematics courses. The Technical Mathematics materials have been used at this school since the second semester of the 1967-68 school year. The results for the past two years are described in the following sections.

Technical Mathematics Course (Spring Semester, 1968).

During the second semester of the 1967-68 school year, one section of Technical Mathematics was offered. The course was taught by a member of the Pius XI High School math department, with help on some days from a teacher aide supplied by the project staff.

Students. The 31 students who volunteered for this class were either juniors or seniors who had completed 5 semesters of college-preparatory math, including one year of algebra, one year of geometry, and one semester of advanced algebra. Volunteers were sought from the group of students who had been experiencing serious difficulties with advanced algebra in the preceding semester. Their lack of fundamental skills was confirmed by their scores on a 20-item pre-test in algebra which was administered at the beginning of the Technical Mathematics course. This 20-item pre-test was an earlier form of the algebra pre-test which was given at the beginning of the 1968-69 school year to the technicians at MATC. A copy of this test is given in Appendix L-1. A distribution of scores and an item analysis for the 31 high school students are given in Appendix L-2. The scores ranged from 0% to 65%; the mean and median scores were 36% and 35%, respectively. Since these students had just completed a third semester of algebra, the low scores could not be attributed to forgetting caused by a long time lapse between math courses and the testing.

Topic-Unit Test Scores. Though the students were initially allowed to proceed completely at their own rates, some minimum standards were eventually set by the teacher. All 31 students completed a minimum of 11 booklets, with a few students completing more. The students were required to take a retest if they did not achieve a minimum acceptable score on the topic-unit test. The mean and median scores on the topic-unit tests for each of the 11 booklets are given in the table below. Retest scores were not included when computing the means and medians.

TOPIC-UNIT TEST SCORES IN TECHNICAL MATHEMATICS
PIUS XI HIGH SCHOOL (JANUARY TO JUNE, 1968)

TOPIC-UNIT	MEAN	MEDIAN
Algebra I: Number Line and Signed Numbers	97%	97%
Algebra II: Non-Fractional Equations I	94%	97%
Algebra III: Non-Fractional Equations II	88%	89%
Algebra IV: Numerical and Literal Fractions	91%	92%
Algebra V: Fractional Roots and Fractional Equations	90%	91%
Algebra VI: Formula Rearrangement	89%	96%
Calculations I: Introduction to Slide Rule	81%	83%
Calculations II: Estimation and Slide-Rule Operations	64%	68%
Graphing I: Reading and Constructing Graphs	91%	92%
Graphing II: Straight Line and Slope	74%	74%
Systems of Equations	79%	80%

In general, the scores on the post-tests were acceptable. Overall, the mean scores were approximately 5 percentage points lower than the mean scores obtained with the technicians at MATC during that year. [Note: When comparing these results with the results obtained by the technicians at MATC, the comparable means and medians for the technicians are given in Chapter 3 in the table entitled "Topic-Unit Test Scores for Technical Mathematics 1 (1967).]

Readministrations of the Algebra Pre-Test. The 20-item algebra pre-test, which had been given in January, 1968, was readministered twice during the semester. Since all items in this test were covered in the first six algebra booklets, the test was readministered in March, 1968, as a general post-test for those six booklets. All 31 students had completed the first six algebra booklets before the test was readministered. The same test was readministered a second time as part of the final exam in June, 1968. Since the students studied slide-rule operations, graphing, and systems of equations between March and June, the readministration in June served somewhat as a measure of retention. The only direct practice on comparable items between March and June occurred in the "Systems of Equations" booklet.

A copy of this test is given in Appendix L-1, as mentioned earlier. The distribution of scores and item analyses for the three separate administrations of the test are given in Appendix L-3. The mean and median for the original pre-test in January were 36% and 35%, respectively. The mean and median for the readministration in March were 89% and 90%, respectively. The mean score for the various sub-sections of the test for each administration are given in the table below.

MEAN SCORES FOR SUB-SECTIONS OF BASIC ALGEBRA TEST
PIUS XI HIGH SCHOOL EXPERIMENTAL CLASS

		JAN., 1968	MAR., 1968	JUNE, 1968
Signed Numbers	(3 items)	69%	97%	90%
Division of Fractions	(1 item)	71%	87%	83%
Non-Fractional Equations	(4 items)	36%	82%	84%
Fractional Equations	(5 items)	26%	84%	69%
Formula Rearrangement	(7 items)	25%	94%	87%

The learning gains were substantial, and the differences in the scores between the two readministrations in March and June show that the amount of forgetting which occurred was slight.

Comparison With Conventional Algebra Classes (Spring Semester, 1968).

The 20-item algebra pre-test was used as a measure of the learning gain from the first six algebra booklets with both the experimental Pius XI class and the technicians at MATC during that year. Substantial gains were obtained with both groups. After 2 months of instruction, the mean and median for the high school class increased from 36% and 35%, respectively, to 89% and

90%, respectively. After only 5 weeks and 2 days of instruction, the mean and median for 402 technicians at MATC increased from 41% and 35%, respectively, to 94% and 95%, respectively. Since the students in the high school class and the technicians at MATC do not represent a cross-section of ability, especially at the upper end, it was difficult to interpret the learning gains and the level of achievement which was obtained. The math department at Pius XI High School agreed to administer the same test to a cross-section of their freshmen and juniors, the two groups who were completing either one or two years of conventional algebra courses. The project staff was interested in this assessment for two reasons: (1) The results could be used as a standard for interpreting the results obtained with the experimental high school class and the technicians at MATC, and (2) The results could be used as a base-rate to assess the amount of forgetting which occurs for technical students between the end of their high school courses and their entry into the technical programs.

The students at Pius XI are divided into five ability-levels ranging from level-1 (highest ability) to level-5 (lowest ability). The algebra test was administered in May, 1968, to 200 freshmen and 137 juniors. A copy of the test is given in Appendix L-1. The distribution of scores and item analyses for each section of freshmen and juniors are given in Appendix L-4. Comparable data for both the pre-test and post-test of the technicians at MATC and the experimental high-school class are also given in Appendix L-4. The post-test occurred after 5 weeks and 2 days of instruction for the technicians at MATC and after 2 months of instruction for the experimental high-school class. The overall median for the freshmen was 30%, with the median for individual sections ranging from 5% to 65%. The overall median for the juniors was 60%, with the median for individual sections ranging from 25% to 85%. The median on the post-test for both the technicians (95%) and the experimental high-school class (90%) was higher than the median for any other high-school section which was tested. The mean scores for each sub-section of the test for the various groups are given in the table below.

MEANS FOR SUB-SECTIONS OF BASIC ALGEBRA TEST

			Pius XI High School Classes		
Sub-Sections		MATC Technicians (Oct., 1967)	Experimental Class (Juniors) (Mar., 1968)	Conventional Classes (Freshmen) (May, 1968)	Conventional Classes (Juniors) (May, 1968)
Signed Numbers	(3 items)	97%	97%	64%	90%
Division of Fractions	(1 item)	92%	87%	48%	88%
Non-Fractional Equations	(4 items)	94%	82%	44%	70%
Fractional Equations	(5 items)	92%	84%	28%	48%
Formula Rearrangement	(7 items)	94%	94%	17%	44%

The table shows that the MATC technicians and the experimental high school class were markedly superior to the conventional freshmen classes in solving either non-fractional or fractional equations and in rearranging formulas. They were also markedly superior to the conventional junior classes in solving non-fractional equations and in rearranging formulas. Since the mean scores for the level-1 juniors on the fractional equations and formula rearrangement sub-sections were 72% and 64% (see Appendix L-4), the technicians and the experimental high school class were even superior to the highest-ability juniors in those two sub-sections of the test.

In order to draw some conclusions from this data, the following factors must be taken into account:

(1) Since Pius XI High School is a private school, its student body is above-average. This fact, coupled with the real concern among the teachers in their math department about the level of achievement of their students, suggests that the results obtained by their students are slightly higher than would be expected from the students in an ordinary public high school.

(2) Some of the low scores might be attributed to the fact that the students knew that this particular test would not figure in their grades for the course.

(3) The test was administered to the technicians at MATC and the experimental high school class immediately after the instruction related to the test was completed. It is possible that the other high school students forgot some of the topics which were tested.

Taking all these factors into account, the following tentative conclusions would seem reasonable:

(1) Either because the choice of topics is too narrow or because of a low level of achievement in those topics which are covered, the algebraic manipulative skills of the high school students seemed quite low. This lack of skill was especially apparent with more complex fractional equations and operations with literal equations. Since such skills are needed by any student who intends to pursue a career in science, technology, or even mathematics itself, we feel that this deficiency is serious. Though "modern math" stresses structure and proof, such a stress is in no way opposed to a mastery of basic manipulative skills nor should it ignore the latter.

(2) Since all of the high school students tested are products of "modern math" instruction, we see no hope for an immediate rise in the level of manipulative skills of students finishing high school until the goals of the high school program are reassessed.

(3) If the level-3 students in this high school are typical of those who enroll in technician-training programs, it appears that the amount of forgetting which occurs between their high school courses and their entry into technician-training programs is negligible. This data certainly supports the decision of the project staff to begin the algebra instruction for technicians from scratch. Most of them are equivalently learning basic principles and skills for the first time.

(4) Given the high level of achievement among the technicians and the "average" students in the experimental high school class, it seems obvious that "average" and even "below average" students can master the basic principles and the basic manipulative skills of algebra. If this mastery level of achievement cannot be matched by the lecture-discussion method, it is clearly a reflection on the inadequacy of that method rather than a reflection on the learning-ability of the students.

Pius XI Technical Mathematics Course (1968-69).

Because of the satisfying results with the experimental class during the spring semester of the previous year, the math department at Pius XI High School decided to offer a Technical Mathematics course to a larger number of students during the 1968-69 school year. The course was offered to juniors and seniors who had already completed two years of college-preparatory math. The following types of students were counseled into the course: (1) those expressing an interest in technical careers, apprenticeships, or other skilled trades, and (2) those who were considered bad risks for further college-preparatory math, but who had expressed an interest in taking another math course. Some of the latter students had intentions of enrolling in a four-year college, but they did not intend to become mathematicians, scientists, or engineers. Though the students in the experimental class the previous year were mainly classified in level-3 (the juniors and seniors are divided into four ability levels), the math department felt that the course could also be offered to level-4 students. Two teachers from the math department of the high school taught the various sections with help from student aides who were selected from the senior class.

Students. The course was offered to 5 sections with a total enrollment of 139 students (104 juniors and 35 seniors). Two of the sections contained level-3 students and three of the sections contained level-4 students. Their IQ's (measured by the Otis Test in the 9th grade) ranged from 92 to 128 with a median of 108. Therefore, the group as a whole was somewhat above average even though they represented the bottom half of their classes in the high school itself.

Pre-Tests in Arithmetic and Algebra. The pre-tests in arithmetic (50 items) and algebra (30 items), which were also administered to the technicians at MATC during that school year, were given to the high school students at the beginning of the semester in September, 1968. A copy of the pre-tests in arithmetic and algebra are given in Appendices B-1 and C-1, respectively. A distribution of scores and an item analysis for the arithmetic pre-test, taken by 138 students, are given in Appendix B-4. A distribution of scores and an item analysis for the algebra pre-test, taken by 139 students, are given in Appendix C-4. The mean and median scores on the arithmetic test were 49% and 46%, respectively; the mean and median scores on the algebra test were 12% and 13%, respectively.

The mean scores for each sub-section of the arithmetic and algebra pre-tests are given in the tables on the next page.

MEAN SCORES FOR SUB-SECTIONS OF ARITHMETIC PRE-TEST
PIUS XI HIGH SCHOOL - SEPTEMBER, 1968

Sub-Section		Mean
Whole Numbers	(4 items)	80%
Decimals	(4 items)	60%
Percents	(6 items)	45%
Number System	(10 items)	50%
Number Sense	(4 items)	62%
Fractions	(22 items)	40%

MEAN SCORES FOR SUB-SECTIONS OF ALGEBRA PRE-TEST
PIUS XI HIGH SCHOOL - SEPTEMBER, 1968

Sub-Section		Mean
Signed Numbers	(6 items)	24%
Powers of 10	(2 items)	4%
Algebraic Fractions	(2 items)	10%
Non-Fractional Equations	(6 items)	31%
Fractional Equations	(6 items)	1%
Formula Rearrangement	(8 items)	2%

Aside from operations with whole numbers, the scores for all other sub-sections in both tests are far from satisfactory.

Topic-Unit Test Scores. Though a few students completed more booklets, all were required to complete 17 booklets during the two-semester course. The amount of content covered did not include much more than had been covered in one semester during the previous year. However, this reduction in speed was necessary because the ability-level of the students was somewhat lower. The mean and median scores for the 17 post-tests are given in the table below. Though retests were given, retest scores were not included when computing the means and medians. The booklets are listed in the order in which they were taught.

TOPIC-UNIT TEST SCORES
FOR TECHNICAL MATHEMATICS COURSE AT PIUS XI HIGH SCHOOL (1968-69)

Topic-Unit	Mean	Median
Algebra I: Number Line and Signed Numbers	92%	95%
Algebra II: Non-Fractional Equations I	91%	93%
Algebra III: Non-Fractional Equations II	91%	93%
Algebra IV: Multiplication and Division of Fractions	89%	92%
Algebra V: Addition, Subtraction and Combined Operations with Fractions	85%	88%
Algebra VI: Fractional Roots and Fractional Equations	84%	86%
Calculations I: Number System and Number Sense	92%	94%
Calculations II: Powers of Ten	90%	92%
Calculations III: Rounding and Rough Estimation	92%	95%
Calculations IV: Introduction to Slide Rule	83%	83%
Algebra VII: Introduction to Graphing	92%	93%
Algebra VIII: Literal Fractions	85%	83%
Algebra IX: Formula Rearrangement	86%	92%
Calculations V: Slide Rule Multiplication and Division	81%	85%
Calculations VI: Slide Rule Powers and Roots	82%	85%
Systems of Equations	75%	77%
Triangles and Trigonometry	80%	83%

The topic-unit test scores were generally satisfying. Though the content was covered considerably slower than it had been with the experimental class during the preceding year, the scores were somewhat lower on most tests. The lower ability-level of many of the students undoubtedly accounts for this fact. Many level-4 students found the course quite challenging, and a retention problem was evident among them. The teachers felt that a partial solution to the retention problem might be some more general type of review sheets or review booklets.

Parallel forms of the topic-unit tests were given as pre-tests for the first two algebra booklets. The mean and median of the pre-test for Algebra I were 51% and 50%, respectively; the mean and median of the pre-test for Algebra II were 30% and 36%, respectively. A decision to give a pre-test for each booklet was abandoned because of the complaints of the students. The students felt that the pre-tests merely reflected a fact which they already knew, namely that they did not possess the skills taught in the booklets. The low pre-test scores for the first two booklets support this fact.

Readministration of the Algebra Pre-Test. The algebra pre-test, which had been administered in September, was readministered as a general post-test for the first nine algebra booklets. The readministration occurred in March after the ninth algebra booklet was completed by 127 students. Little formal review was given before the test. The original mean and median for these 127 students were 13% and 13%, respectively; the mean and median on the readministration were 68% and 70%, respectively. The distribution of scores and item analysis for each administration are given in Appendix C-5. The mean scores for each sub-section of the two administrations of the test are given in the table below:

MEAN SCORES FOR SUB-SECTIONS OF ALGEBRA PRE-TEST
PIUS XI HIGH SCHOOL - SEPTEMBER, 1968 AND MARCH, 1969

Sub-Sections		Mean (Sept., 1968)	Mean (March, 1969)
Signed Numbers	(6 items)	25%	85%
Powers of 10	(2 items)	4%	62%
Algebraic Fractions	(2 items)	10%	68%
Non-Fractional Equations	(6 items)	32%	76%
Fractional Equations	(6 items)	1%	47%
Formula Rearrangement	(8 items)	2%	67%

In spite of the fact that the gains were substantial, there was still considerable room for improvement. The students would undoubtedly have scored higher in March if the readministration had been preceded by some review sessions.

Though the 70% median in March compared favorably with the 60% median achieved on a comparable test by the juniors finishing a second year of college-preparatory algebra during the preceding year, it was considerably lower than the 90% median achieved by the experimental class during the preceding year. There are various factors which probably contributed to the lower score:

(1) The lower ability and retention problem of the level-4 students.

(2) The fact that the algebra instruction was spread over almost one and one-half semesters, whereas with the experimental class in the preceding year it had been completed in a concentrated period of 2 months.

(3) The fact that the students in the experimental class during the preceding year had just completed a third semester of college-preparatory algebra, whereas those in 1968-69 had completed only two semesters of algebra with at least a one-year time lapse between their previous algebra instruction and the Technical Mathematics course.

Arithmetic Skills of Entering Freshmen at Pius XI.

The 50-item arithmetic test which was given at the beginning of the Technical Mathematics course was given to a cross-section of the entering freshmen at the beginning of the 1968-69 school year. This assessment of arithmetic skills was conducted for two reasons. First, it gives a gross measure of the success of math instruction in elementary schools. Second, it gives some indication to the high school math teachers of the amount of remedial work in arithmetic which should be included in the high school math program.

A copy of the arithmetic test is given in Appendix B-1. The distribution of scores and the item analysis for the 127 freshmen who took the test is given in Appendix B-5.

There were 18 mathematics sections in the freshmen class in 1968-69. Of these 18 sections, there were 4 sections of level-1, 4 sections of level-2, 8 combined sections of levels 3 and 4, and 2 sections of level-5. Of these 18 sections, the test was administered to one section each of levels 1, 2, and 5, and to one combined section of levels 3 and 4. The overall mean and median for each of these four sections is given in the table below:

MEAN AND MEDIAN SCORES ON ARITHMETIC PRE-TEST
FOR DIFFERENT ABILITY LEVELS OF ENTERING FRESHMEN
PIUS XI HIGH SCHOOL (1968-69)

Ability Level	N	Mean	Median
Level 1	32	79%	80%
Level 2	31	68%	68%
Level 3 & 4	32	47%	50%
Level 5	32	24%	22%
Overall	127	54%	60%

The means for each of the sub-sections of the test for each level are given in the following table:

MEAN SCORES FOR SUB-SECTIONS OF ARITHMETIC PRE-TEST
FOR DIFFERENT ABILITY LEVELS OF ENTERING FRESHMEN
PIUS XI HIGH SCHOOL (1968-69)

Sub-Sections		Level 1	Level 2	Level 3 & 4	Level 5
Whole Numbers	(4 items)	88%	84%	84%	66%
Decimals	(4 items)	88%	79%	58%	30%
Percents	(6 items)	91%	78%	42%	13%
Number System	(10 items)	72%	65%	50%	18%
Number Sense	(4 items)	79%	65%	51%	30%
Fractions	(22 items)	76%	61%	38%	20%

Except for operations with whole numbers, the achievement across the various levels drops quite rapidly. There seems to be considerable room for improvement at all levels in the number system, number sense, and fraction items. Unfortunately, the content of most high school math courses does not include much formal review of these topics. This lack of formal review is reflected by the fact that the median score on this test for the level-3 and level-4 juniors in the Technical Mathematics course was 46%, which is 4% lower than the median score for the level-3 and level-4 freshmen.

Discussion of the Results at Pius XI High School.

Both the math department at Pius XI High School and the project staff have been pleased with the results obtained in the experimental classes. A new course has been offered to a group of juniors and seniors for whom no other math course is available. Even though these students represent the bottom-half of their respective classes in terms of ability and achievement, their achievement in the math class has been at a fairly high level. In fact, since the high school students are not as mature or as motivated as the technicians at MATC, the fact that the high school students' performance has not been appreciably below that of the technicians has been most encouraging.

As a result of the success with a first year of Technical Mathematics, a second year of Technical Mathematics will be tried at Pius XI during the 1969-70 school year. The prerequisite for the second-year course will be completion of the first year of Technical Mathematics. In general, only those students will be allowed to enroll who maintained a 90% average during the first year. The topics covered in the second-year course will be mainly those included in Technical Mathematics 2 for the technicians at MATC. The tentative schedule of topics by quarters of the school year is:

Quarter 1 - Vectors
General Angles
Oblique Triangles
Introduction to Logarithms

Quarter 2 - Quadratic Equations
Radicals and Radical Equations
Graphing II: Straight Line and Slope

Quarter 3 - Complex Numbers
Sine-Wave Analysis
Geometry and Applied Trig
Further Trig Topics

Quarter 4 - Common and Natural Logarithms
Laws of Logarithms and Logarithmic Formulas
Technical Measurement

Much of the success of the experimental program has to be attributed to the spirit with which it has been approached by Pius XI High School. The math department has been seriously interested in its success. The two teachers involved have been very cooperative and flexible, and by means of their organizational skills they have developed a system of instruction which works in their high school setting. Probably one of the key factors has been the interest of the teachers in the learning produced in their students rather than in the mere coverage of topics. This interest in learning has enabled them to develop a pace in the course which parallels the students' ability to learn.

The teachers have made the following comments about their first full year of experience with the experimental classes:

(1) More learning occurs than in the traditional method with this level of student.

(2) The individual problems of each student can be met.

(3) The discipline of reading and learning for himself is an invaluable experience for the student.

(4) There is a definite pattern of student reaction in terms of the four quarters of the school year. It is:

Quarter 1 - very enthusiastic
Quarter 2 - bored, does not see how the work is tied together
Quarter 3 - enthusiasm begins to pick up again
Quarter 4 - really sees value of the course

(5) Retention is a problem. Either more review should be included in the booklets themselves, or some additional review materials should be provided. Perhaps each booklet should contain some review sheets at the end of it. Furthermore, the attitudes of the students towards long-range retention must be improved. Many of them approach each individual programmed unit as if it were a self-contained entity. They do not seem to realize that the skills learned in one booklet will be needed in later booklets in the course.

West Division High School

West Division High School is a core-area school in the Milwaukee Public School System. An experimental class was begun during the second semester of the 1967-68 school year. The teacher was a member of the math department of the high school; he was assisted by a teacher aide who was supplied by the project staff. The class, which was offered as an alternate for general math, included sophomores, juniors, and seniors. Since the decision to offer the experimental class was made shortly before the beginning of the second semester, the school had no opportunity to select students for it on any rational basis. The class became somewhat of a dumping ground for problem students, and the results were not very satisfactory. Therefore, we will only report the results obtained during the 1968-69 school year.

Technical Mathematics Course (1968-69).

During the 1968-69 school year, one section of Technical Mathematics was offered. The course was taught by the head of the math department at the high school, with help from a teacher aide supplied by the project staff.

Students. The 25 students in the Technical Mathematics class included sophomores, juniors, and seniors. In order to assure a reasonable test of the materials, the students were selected by a guidance counselor on the basis of their having a fair chance of success in the course. Many had already taken an algebra course; most of those who had done so had failed the algebra course. Their IQ's ranged from 76 to 112, with a median of 100. As a group, they were somewhat above average in the school.

Pre-Tests in Arithmetic and Algebra. The pre-tests in arithmetic (50 items) and algebra (30 items) which were administered to the technicians at MATC during that school year were given to all students at the beginning of the course. Copies of the arithmetic and algebra tests are given in Appendices B-1 and C-1, respectively. A distribution of scores and an item analysis for each test are given in Appendices B-6 and C-6, respectively.

The mean and median scores on the arithmetic test were 33% and 34%, respectively. The mean and median scores on the algebra test were 20% and 17%, respectively. The mean scores for each sub-section of each test are given in the table below and in the table on the next page:

MEAN SCORES FOR SUB-SECTIONS OF ARITHMETIC PRE-TEST
WEST DIVISION HIGH SCHOOL - SEPTEMBER, 1968

Sub-Section		Mean
Whole Numbers	(4 items)	68%
Decimals	(4 items)	42%
Percents	(6 items)	30%
Number System	(10 items)	30%
Number Sense	(4 items)	44%
Fractions	(22 items)	27%

MEAN SCORES FOR SUB-SECTIONS OF ALGEBRA PRE-TEST
WEST DIVISION HIGH SCHOOL - SEPTEMBER, 1968

Sub-Section		Mean
Signed Numbers	(6 items)	41%
Powers of Ten	(2 items)	15%
Algebraic Fractions	(2 items)	12%
Non-Fractional Equations	(6 items)	47%
Fractional Equations	(6 items)	1%
Formula Rearrangement	(8 items)	2%

Topic-Unit Test Scores. Because of the heterogeneous ability of the students and the fact that absenteeism is a problem in the school, the students were allowed to proceed at their own rates. Occasionally, some students were prodded to make a more serious effort; this prodding was more or less successful. Therefore, there was a wide range in the number of booklets completed by the students. This range was increased by the fact that 8 of the students were only in the class during the first semester. The number of booklets completed by these 8 students is given in the following table:

Number of Booklets Completed	Number of Students
1	2
2	2
6	1
7	3

Of these 8 students, 2 were dropped because they refused to make any effort, 2 who made little effort were retained until the end of the first semester, and 4 who made satisfactory progress decided against enrolling for the second semester. The number of booklets completed by the 17 students who completed both semesters is given in the following table:

Number of Booklets Completed	Number of Students
8	2
9	2
11	2
12	3
13	3
14	1
15	1
17	1
18	1
19	1

The table below contains the mean and median scores for the booklets completed by 10 or more students. Though much retesting was done, the retest scores were not included when computing the means and medians. Since the number of students completing each booklet varied, this number is also included in the table.

TOPIC-UNIT TEST SCORES IN TECHNICAL MATHEMATICS
WEST DIVISION HIGH SCHOOL (1968-69)

TOPIC-UNIT	N	MEAN	MEDIAN
Algebra I: Number Line and Signed Numbers	25	87%	94%
Algebra II: Non-Fractional Equations I	23	79%	89%
Algebra III: Non-Fractional Equations II	21	88%	89%
Algebra IV: Multiplication and Division of Fractions	21	78%	77%
Algebra V: Addition, Subtraction, and Combined Operations with Fractions	21	70%	73%
Algebra VI: Fractional Roots and Fractional Equations	19	75%	82%
Algebra VII: Introduction to Graphing	17	87%	87%
Algebra VIII: Literal Fractions	17	74%	83%
Algebra IX: Formula Rearrangement	14	80%	84%
Calculations I: Number System and Number Sense	18	81%	85%
Calculations II: Powers of Ten	12	88%	90%

The results were lower than those achieved by the technicians at MATC and the experimental classes at Pius XI High School. If a student completed more booklets than those listed above, he did so in the following order:

Calculations III: Rounding and Rough Estimation
Calculations IV: Introduction to Slide Rule
Calculations V: Slide Rule Multiplication and Division
Calculations VI: Slide Rule Powers and Roots
Systems of Equations
Quadratic Equations
Radicals and Radical Equations
Straight Line and Slope

Readministrations of the Algebra Pre-Test. The algebra pre-test, which had been given in September, 1968, was readministered twice during the school year. The first readministration occurred in April, 1969. At that time, the test was used as a general test of improvement resulting from the first nine algebra booklets. Unfortunately, some of the students had not completed all nine booklets by that time. The test was given without any warning and without any review in order to get as pure a measure of retention as possible. The second readministration occurred in June, 1969, when a parallel form of the pre-test was incorporated in the final exam. There was some formal reviewing before the final exam. A copy of the test is given in Appendix C-1.

A distribution of scores and an item analysis for each of the three administrations is given in Appendix C-7. Since only 15 students took all three tests, only their scores are included in the analyses. For these 15 students, the mean and median in September were 23% and 27%, respectively, the mean and median in April were 64% and 73%, respectively; the mean and median in June were 83% and 90%, respectively. The mean score for each sub-section of the test for each of the three administrations is given in the table below:

MEAN SCORES FOR SUB-SECTIONS OF ALGEBRA PRE-TEST
ADMINISTERED AT THREE DIFFERENT TIMES
WEST DIVISION HIGH SCHOOL (1968-69)

Sub-Sections		Sept., 1968	April, 1969	June, 1969
Signed Numbers	(6 items)	47%	80%	94%
Powers of Ten	(2 items)	20%	40%	73%
Algebraic Fractions	(2 items)	17%	57%	97%
Non-Fractional Equations	(6 items)	53%	72%	84%
Fractional Equations	(6 items)	2%	44%	79%
Formula Rearrangement	(8 items)	2%	68%	74%

The learning gains were substantial, and the formal review before the final exam in June had a definite beneficial effect.

Readministration of the Arithmetic Pre-Test. The arithmetic pre-test, which had been administered in September, 1968, was readministered on a regular class day in June, 1969. There was no formal review before the test was readministered. A copy of the test is given in Appendix B-1. Distributions of scores and item analyses for the two administrations are given in Appendix B-7. Since only 15 students took both tests, only their scores are included in the analysis. The mean and median in September were 37% and 38%, respectively; the mean and median in June were 65% and 68%, respectively. The mean scores for each of the sub-sections of the test are given in the table below:

MEAN SCORES FOR SUB-SECTIONS OF ARITHMETIC PRE-TEST
ADMINISTERED AT TWO DIFFERENT TIMES
WEST DIVISION HIGH SCHOOL (1968-69)

Sub-Sections		Sept., 1968	June, 1969	Gains
Whole Numbers	(4 items)	73%	93%	+20%
Decimals	(4 items)	42%	67%	+25%
Percents	(6 items)	36%	48%	+12%
Number System	(10 items)	32%	67%	+35%
Number Sense	(4 items)	45%	75%	+30%
Fractions	(22 items)	30%	62%	+32%

Though the learning gains were substantial, the final level of performance still leaves considerable room for improvement. The greatest gains were obtained with the number system, number sense, and fractions items, the three topics which were emphasized in the course. It is interesting to note, however, that gains occurred with the whole number, decimal, and percent items, even though these topics were not specifically taught.

Comparison with Conventional High School Algebra Classes. When the algebra pre-test was readministered to the students in the Technical Mathematics class at West Division High School in April, it was also administered to two first-year, conventional algebra classes at the same school. One of these classes was taking the first semester of Algebra I; the other was taking the second semester of Algebra I. In order to avoid an unfair advantage for any class, the test was given without warning or review in all three classes. A copy of the test is given in Appendix C-1. A distribution of scores and an item analysis for each class is given in Appendix C-8. The mean and median for 16 students in the Technical Mathematics class were 64% and 73%, respectively. The mean and median for 19 students in Algebra I (first semester) were 22% and 20%, respectively. The mean and median for 24 students in Algebra I (second semester) were 38% and 35%, respectively. The mean score for each of the sub-sections of the test for the three classes is given in the table below:

MEAN SCORES FOR SUB-SECTIONS OF ALGEBRA PRE-TEST
FOR TECHNICAL MATHEMATICS CLASS AND TWO CONVENTIONAL ALGEBRA CLASSES
WEST DIVISION HIGH SCHOOL (APRIL, 1969)

Sub-Sections		Technical Mathematics	Algebra I (First Semester)	Algebra I (Second Semester)
Signed Numbers	(6 items)	80%	54%	59%
Powers of Ten	(2 items)	40%	3%	27%
Algebraic Fractions	(2 items)	57%	13%	27%
Non-Fractional Equations	(6 items)	72%	46%	62%
Fractional Equations	(6 items)	44%	1%	31%
Formula Rearrangement	(8 items)	68%	1%	18%

At least in terms of the algebraic manipulative skills assessed by this particular test, the students in the Technical Mathematics class were superior. In fact, they were superior to each of the other two classes on each subsection of the test.

Discussion of the Results at West Division High School. The results achieved by the students at West Division High School did not equal those obtained by the experimental classes at Pius XI High School. The majority of the students at West Division did not complete as many booklets, and their scores on the booklets completed were lower. Of course, this difference could have been predicted for two reasons: (1) The ability-level of the students at West Division was generally much lower than the ability-level of the students at Pius XI, and (2) All of the students at Pius XI had completed two years of college-preparatory math courses, whereas most of the students at West Division had not completed any college-preparatory math courses.

The math teacher who handled the class at West Division was quite enthusiastic about the results achieved. He pointed out various positive factors:

(1) The students learned more than they would have learned in a conventional math class.

(2) Only 4 of 25 students refused to make some type of an effort, and though some of the others did not progress as far as they could have, the majority of the students worked diligently throughout the year. In fact, the effort made by some of the slower learners was quite impressive.

(3) The use of programmed materials and self-pacing is an ideal way to cope with absenteeism, which is a problem with some of the students.

(4) The comparison with conventional algebra classes on a common exam showed that the students in the Technical Mathematics class were more than competitive with the students in the regular algebra classes, at least in terms of the algebraic skills which were assessed.

In fact, the math teacher, who was also the head of the mathematics department at West Division, suggested that Technical Mathematics should replace all of the general math classes at the school.

General Discussion of Results With Experimental High School Classes

Success of the Experimental Classes.

The results in the experimental high school classes were generally satisfying. The level of achievement of the students was reasonably high, in spite of the fact that the learning materials were not specifically written for high school students. The students in the experimental classes at both Pius XI and West Division High Schools compared very favorably to students in the conventional algebra classes. This favorable comparison was obtained even though the ability-level of the students in the experimental classes was usually not as high as the ability-level of the students in the conventional classes. Though these comparisons were based on items which assessed the specific skills which were taught in the experimental classes, most mathematics teachers would admit that the development of these skills should be a part of the goal of any algebra instruction.

The success of the experimental classes is related to many factors. The programmed materials communicate better with the students than lectures or conventional textbooks do. The use of programmed materials gives the teachers an opportunity to come to grips with the learning difficulties of individual students. The students can proceed at their own rates, and the teachers can insist on a high level of performance in one topic before a student progresses to the next topic. The use of programmed materials offers a mechanism for coping with the absenteeism which is bound to occur. A mechanism of this type is especially useful with the type of student for whom absenteeism is somewhat a chronic problem. The use of programmed materials also tends to reduce "discipline" problems, although it certainly does not eliminate them.

Teacher and Student Reaction.

The reaction of the high school teachers to the experimental classes has been quite positive. They have been impressed with the students' level of achievement on relevant skills, and they have also been impressed with the results of the comparisons with conventional algebra classes. Furthermore, they realize that these classes, with their use of programmed materials and tutoring, have increased the number of students to whom a relevant math course can be offered. For example, there is no alternate math course at Pius XI for most students who enroll in the Technical Mathematics course, since they are not allowed to enroll in further college-preparatory math courses and general math is not offered. Similarly, general math is the only alternative for most students in the Technical Mathematics course at West Division, and general math courses are usually not very relevant. The high school teachers seem to enjoy the opportunity to work with students on an individual basis, and since a fair amount of tutoring is needed with many students, the teachers soon realize that their efforts are an integral and essential part of the learning system. Furthermore, they can afford to be quite demanding with the students, and even raise the level of aspiration of the students, since the learning system has demonstrated that a high achievement level can be maintained.

The student reaction has also been quite positive. Their positive attitude is probably reflected best in their motivation level, which has generally been quite good. In fact, the motivation level of the students has been high enough to make members of the project staff wonder whether motivation among high school students is really as serious a problem as many educators claim. It seems plausible that student motivation is related to their success. For many of the students, their success experience in the experimental classes contrasts markedly with their lack of success in conventional math courses. Perhaps many high school students become demotivated simply because the conventional method of instruction does not communicate with them. It is not sensible to expect students to continue to make an effort when their efforts are unsuccessful.

Possible Improvements for High School Students.

The high school classes would probably be even more successful if the programmed materials were written specifically for high school students. This rewriting is more necessary for slower students who are learning algebra for the first time. For them, materials with shorter frames would clearly be better. Also, various improvements could be made for all high school students, even those who had previously studied algebra. Daily tests should be written to cover shorter assignments. Some review materials should be developed to offset the forgetting which occurs with many students. And probably better results could be obtained if a more flexible "learning center" approach were developed for the instruction. The use of a "learning center" approach has not been used because it presupposes three conditions: (1) a large number of students, (2) more than one class section scheduled at the same time, and (3) fairly flexible physical facilities. Ordinarily, these three conditions cannot be met in the high schools, or at least they have not been up to the present time.

Mathematical Skills of High School Students.

Enough assessment of the mathematical skills of high school students has been done in conjunction with the experimental high school classes to obtain at least a rough estimate of the achievement level of high school students in some basic skills. The arithmetic skills of high school students were assessed in a number of classes by means of a 50-item arithmetic test (See Appendix B) during the 1968-69 school year. The median scores for the experimental classes at Pius XI and West Division High Schools at the beginning of the school year were 46% and 34%, respectively. The median scores for four beginning freshmen classes at Pius XI High School were 80% (level-1), 68% (level-2), 50% (level-3 and level-4 combined), and 22% (level-5). Except for the level-1, and possibly the level-2, freshmen at Pius XI, none of these scores were very acceptable. The 46% median for the experimental class at Pius XI was especially disturbing since these level-3 and level-4 juniors and seniors had already completed two years of college-preparatory math courses. Since this 46% median for level-3 and level-4 juniors is lower than the 50% median for level-3 and level-4 freshmen, it seems obvious that the high school curriculum makes little provision for remedying arithmetic deficiencies.

The algebraic skills of high school students were assessed by a 20-item algebra test (See Appendix L) during the 1967-68 school year and by a 30-item algebra test (See Appendix C) during the 1968-69 school year. During the 1967-68 school year, all of the testing was done at Pius XI High School. The median score of the experimental class when the course began in January was 35%; these students had just completed a fifth semester of college-preparatory math. The overall median for a cross-section of the conventional freshmen classes at the end of the school year in May was 30%, with the medians for the six individual sections ranging from 5% to 65%. The overall median for a cross-section of the conventional junior classes at the end of the school year in May was 60%, with the median for the five individual sections ranging from 25% to 85%. During the 1968-69 school year, the testing was done at both Pius XI and West Division High Schools. The scores for the experimental classes at Pius XI and West Division at the beginning of the school year were 13% and 17%, respectively. The median scores for two conventional algebra classes at West Division in April were 20% for a first-semester Algebra I section and 35% for a second-semester Algebra I class. In general, the scores were not high. Since the student body at Pius XI High School is clearly above average, it was somewhat disturbing that only the highest ability (level-1) sections of freshmen and juniors obtained a median score higher than 50% at the end of the 1967-68 school year. When assessing the scores of the junior class, it must be remembered that only approximately the top half of that already select class are allowed to enroll in a third-year college-preparatory math course.

There is a suggestion in the high school data that conventional mathematics instruction is only communicating with the top 25% or 30% of the students. The great majority of the students are deficient in arithmetic topics like operations with decimal numbers, percents, number system, number sense, and fractions. They are also deficient in algebraic topics like algebraic fractions, fractional equations, and formula rearrangement. Many are even deficient in very elementary algebraic topics like signed numbers and non-fractional equations. There seems to be no question that the majority of high school students can learn these mathematical skills to a significantly higher level. The data from the experimental classes attests to this fact.

Meaning of the Low Entry Skills of MATC Technicians.

During the 1968-69 school year, the median scores on the arithmetic and algebra pre-tests for the entering technical students at MATC were 66% and 30%, respectively. The low scores can be attributed either to a lack of original learning, a large amount of forgetting, or some combination of the two. Since students who have never learned must be treated differently than students who have learned but forgotten, the project staff had to make a judgment whether the low scores should be attributed more to a lack of learning or more to forgetting. The staff members felt that most of the students suffered from a lack of original learning. Consequently, many elementary topics in both arithmetic and algebra were included as an integral part of the Technical Mathematics instruction. This decision is supported by the high school data. In fact, it seems logical to say that, if anyone wants to claim that these low scores simply reflect a large amount of forgetting, the burden of proof is now on him to show that the type of student who enrolls in technician training has these skills to a much higher degree at some time. It also seems obvious that the minimal mathematics requirement (one year of algebra and one year of geometry) for entry into the technical programs is relatively meaningless. The median score on an algebra test for students completing one year of conventional algebra at Pius XI and West Division were only 30% and 35%, respectively. And these low algebra scores do not take into account the fact that many of the same students also have deficiencies in basic arithmetic skills.

Non-Science Orientation of High School Mathematics Content.

The amazing thing about the high school mathematics content is the fact that it apparently does not emphasize the topics and skills which students need to learn basic science and technology. The majority of the students are deficient in topics like fractions, fractional equations, and formula rearrangement, even though skills in these topics are essential for any mathematical approach to science and technology. If students cannot handle literal fractions or rearrange literal expressions, it seems inconceivable that they could follow even some of the simpler derivations in science or technical courses. Many of the students are even deficient in arithmetic skills, with no apparent provision in the high school content for remedying many of the deficiencies. It is also significant that only 34% of the entering technical students have had any introduction to the use of a slide rule.

Whatever the high school mathematics content does emphasize, it certainly cannot claim that it is successful, or even interested, in preparing the majority of the students for elementary science and technical courses. It is no wonder, therefore, that enrollment in high school physics courses is so low. It is no wonder that some physics teachers are even attempting to develop a physics course in which no mathematical skills are needed. If the mathematical skills needed for elementary science were unattainable for most students, a non-mathematical approach to physics instruction would be necessary. But the results in the experimental math classes suggests that the unattainability of these math skills has been seriously overestimated. In fact, it seems that a high percentage of students could learn these skills at a relatively high level. Our culture is built on science and technology. More and more jobs in our culture require an understanding of basic science and technology. Anyone in our culture who does not have a rudimentary knowledge of the principles of science and technology could hardly be called "liberally" educated. In a culture such as this, it is difficult to justify a high school mathematics curriculum which does not support scientific and technical education for a higher percentage of students.

Higher-Ability Students and Programmed Materials.

When the programmed materials have been used in high schools, they have always been used with students whom the schools view as either learning or discipline problems. The materials have never been tried with higher-ability, well-motivated students. High school teachers usually say that they would not be "good" for faster learners. By not being "good" for faster learners,

they probably mean either that the content is too restricted or that the faster learners would be bored by the many detailed steps included in the instruction. They obviously cannot mean that faster learners would not learn from programmed materials, especially since it has been demonstrated that slower learners do learn from them.

Obviously, the content is too restricted for the faster learner, and a full course in algebra, for example, for the faster learner should contain more topics. However, it would be interesting to see how fast a group of faster learners could proceed through the content which the materials contain. It would also be interesting to see at how high a level they would perform and how much tutoring they would require. It would definitely be interesting to see how "bored" they would be by the use of programmed materials in comparison with conventional instruction, which itself is not always a "non-boring" experience. If the faster learners, for example, could learn basic skills with so little tutoring that they would not have to attend class, they might conceivably prefer programmed instruction to conventional methods. Hopefully, some school will offer the opportunity to answer these questions some day.

CHAPTER 5

GENERAL IMPLICATIONS AND FUTURE DIRECTIONS

In this chapter, the accomplishments during the four-year history of the project will be summarized and discussed. This discussion will naturally lead into a broader discussion of the implications of the project for mathematics education and for education in general. After outlining the many future directions which the project can take, the chapter will conclude with a brief summary of the general significance of the project.

Accomplishments of the Project

A system of instruction has been developed to teach the basic mathematical skills needed for elementary science and technology. This system of instruction is a radical departure from conventional instruction in terms of both the content taught and the method of instruction. Designed for average and below-average learners, the major components of the system are programmed materials, continual diagnostic assessment, and tutoring. The system has been designed to maximize the opportunity for interactions between the teacher and individual students so that the teacher can gain maximum control over the learning process of each student. Within the context of the system, each student's progress can be either paced or self-paced. Also, the very nature of the system provides a mechanism for coping with absenteeism.

Technical Mathematics at MATC.

The system was originally developed for the Technical Mathematics course for industrial technicians at the Milwaukee Area Technical College. During its four-year development for that two-semester course, the system has gradually evolved into the use of a Learning Center which offers separate treatments for fast, regular, and slow learners. The operation of the Learning Center has become more efficient and economical because of the use of teacher aides and clerical personnel. The following three goals have been achieved:

(1) The content of the course has been made more relevant to the needs of industrial technicians.

(2) The dropout rate in the mathematics course has been cut approximately in half.

(3) The achievement level of the students has been significantly increased.

The reaction of the students to the system of instruction has been overwhelmingly positive. The reaction of the teachers has become progressively more positive during the course of the project, but the enthusiasm of some teachers does not match that of the students.

Experimental High School Classes.

Though not specifically designed for high school students, the materials have been used on an experimental basis in various high schools in the Milwaukee area. The materials have been used either as a replacement for general math or as a pre-algebra course. Data has been reported from Pius XI High School, where a two-year Technical Mathematics sequence is now offered to various sections of juniors and seniors, and from West Division High School, where a one-year course is offered to one combined section of sophomores, juniors, and seniors. The results in the high school courses have been generally satisfying. Comparisons with conventional algebra classes on a common exam revealed that the students in the experimental classes were more than competitive with most conventionally-taught students, in spite of the fact that their ability level was generally lower than the ability level of students in the conventional classes. The reaction of students and teachers in the experimental high school classes was generally positive.

Other Uses of the Materials.

Besides their use for the technicians at MATC and in high schools, the learning materials have been used on an experimental basis in various classes both within MATC and elsewhere. Outside of MATC, the materials have been used in a few other technical schools in the State of Wisconsin either for the Technical Mathematics course itself or in a pre-technical program. Within MATC, they have been used in some apprentice programs, in some trade programs for adults, and in a junior college developmental program. Though not specifically designed for these latter courses, the materials were used because the teachers asked to use them. When used in courses other than Technical Mathematics, the teachers were ordinarily interested in using the basic algebra and calculation booklets. No matter where the materials have been used, the general reaction of the teachers has been positive.

Success and Limitations.

There are many reasons for the success of the system of instruction developed by the project staff. The system is highly organized. It provides for interactions between the teacher and individual students so that the teacher can gain control over the learning process of each student. It has objectives which are clear both to the teacher and the student, and only these objectives are assessed by test items. It uses learning materials which take into account what is known or being discovered about the learning process of the slower learner. Since the students have a chance at success and some success experience, their motivation level remains comparatively high. And based on the fact that the students can be successful, teachers can afford to deliberately raise the students' level of aspiration.

Since the learning materials have been specifically written for the technical students at MATC, they have been most successful with this group or with students who have comparable entry skills in algebra and geometry. In general, students with this background progress through the materials at a faster rate, and they maintain a higher level of performance. When the materials have been used with high school students who are learning algebra for the first time, the students involved have always been average or slower learners. Though this group has learned from the materials, the efficiency of their learning could be increased by the development of a set of materials written specifically for them. With no evidence to support their view, members of the project staff feel that the materials in their present form would effectively communicate with the faster learners in high schools, even if they were learning algebra for the first time.

There is a general opinion in our society, both within and outside of the school system, that average and below-average students cannot learn mathematics. The results achieved by the project suggest that this opinion is false. It seems that many average and below-average learners can master even complex mathematical skills if they are instructed properly. In fact, it seems that a system of instruction like the one developed by the project staff would communicate with as many as 70% or 75% of the students in our schools. A more refined system would probably communicate with an even greater number.

Mathematics Education

There are many indications that mathematics education in the elementary and secondary schools is not highly successful. Vocational and technical training institutions constantly complain about the lack of relevant mathematical skills in their entering students. College math teachers in four-year institutions also complain about a lack of mathematical skills in their entering students. In fact, most four-year colleges are forced to offer non-credit remedial math courses to a large percent of their students. Science teachers, at both the high school and college levels, find many students who are mathematically unequipped for their courses. In this section, we will discuss some general problems in mathematics education under the following headings: (1) the general need for mathematical skills in our society, (2) the fact that this need is not being fulfilled, (3) why this need is not being fulfilled, with special emphasis on the attitudes and beliefs of math educators, (4) how the average student is abused, (5) the need for a new minimum math curriculum, and (6) the need for new methods of math instruction. Though much of what will be said is applicable to math education at all levels, special attention will be given to math education at the secondary school level.

General Need for Mathematical Skills.

In order to evaluate the goals and success of mathematics education as it currently exists in our educational system, the gross facts about the need for mathematical skills in our society must be kept in mind. Without the perspective of these gross facts, any meaningful evaluation is impossible. One fact cannot be emphasized enough. That is, the percent of students who need high-level mathematical skills for their professional careers is quite small. Probably less than 5% of the students in our society become professional mathematicians or high-level scientists or engineers. Not only does this small segment of the job market seem to be adequately filled at the present time, but there is no reason to believe that it will significantly exceed 5% in the immediate future. However, as our society becomes increasingly scientific and technical, the need for math skills among the other 95% of the students is growing on two fronts. One front is the job market. With the number of unskilled jobs diminishing, the job market includes a growing number of skilled and technical jobs which require math skills both on the job and in job training. The second front is liberal education. To be liberally educated in our society, a student must understand the basic principles of science and technology upon which this society is based. He cannot learn these basic principles beyond a mere descriptive level without the basic math skills which this learning requires.

An Unfulfilled Need.

It seems obvious that the math curriculum in our schools should reflect the mathematical needs of the majority of the students. Therefore, it seems obvious that the math content in our high schools should, as a minimum, prepare the students for courses in elementary science or technology. Unfortunately, this preparation does not occur. The high school data we have presented attests to this fact. Aside from the brightest students, the students were uniformly deficient in topics like fractions, fractional equations, and formula rearrangement. And though fluency with numbers is required in most science or technical courses, only one out of three entering technical students has even been introduced to the slide rule. Judging from the high school data, most students who complete only one year of high school algebra are not prepared to take basic science or technical courses. Many students who complete two years of high school algebra are not prepared to take basic science or technical courses. It is small wonder that the percent of students taking physics in high school is quite low. The rest are simply unprepared to do so. They are also unprepared to take most technical courses.

In comparison with the needs of our society, many students graduate from high school with an inadequate mathematical preparation. Their preparation is inadequate because the content which is taught ignores too many topics which are needed in science or technical courses, and conventional methods of instruction apparently communicate with only the top quarter or third of the students. Consequently, many high school graduates do not have the mathematical skills which they need for their job training or further education. By failing to meet the needs of these students, the school system is also failing to meet the needs of our society.

Why the Need is Unfulfilled.

There are many reasons why the school system is not fulfilling our society's needs for an adequate mathematical preparation in a higher percentage of students. The math curriculum in the school system is dominated by the thinking of high-level mathematicians. The present method of instruction is inadequate because it fails to communicate with the majority of the students. The overall effectiveness of the system is unassessed. Though teachers realize that many students are not learning, many of them feel that instructional improvements of any significance are almost impossible. And criticisms of the content and effectiveness of the math instruction are too vague, with no concrete suggestions as to how either the content or method of instruction can be improved. We will discuss these reasons in this section and the subsequent ones.

Math Curriculum. Mathematics education in our school system is dominated by the thinking of high-level mathematicians. This fact is obvious from an examination of the "modern math" curriculum. Many of the goals and much of the content of this curriculum are much more relevant, necessary, and interesting to the professional mathematician than they are to the majority of the students. For the average student, emphasis on such lofty goals as "thinking like a mathematician," "creative discovery," "the ability to prove," and "an insight into the structure of axiomatic-deductive systems" are absurd. For the average student, content such as "number systems other than base-10," "the real number system," "inequalities," and "set theory" are neither necessary nor useful. For the average student, a deemphasis on numerical fluency, manipulative skills, and more science-oriented topics is a genuine disservice. The "modern math" curriculum is basically designed to prepare students for higher-level mathematics training. The majority of the students in our society do not need this training and they are not interested in it. Despite the fact that the "modern math" curriculum in many ways ignores the needs of the majority of our students, that curriculum is virtually unchallenged. It is unchallenged by math educators because they reinforce each other; it is virtually unchallenged by anyone else because most other people do not feel competent to challenge it.

Math educators in the school system have to free themselves from the domination of high-level mathematicians and begin to seriously examine the real-world facts about the math needs of the majority of the students. The vast majority of students have neither the ability nor inclination to become high-level mathematicians. The vast majority of people who use mathematics in their personal lives or careers use it in applied situations. Since a high percentage of students are not learning many fundamental math skills, to speak in terms of general goals like "thinking like a mathematician" or "creative discovery" for these students is ridiculous. The vast majority of students, even many of the brighter ones, are bored with proofs and insights into axiomatic-deductive systems. Furthermore, an emphasis on "proof" and "understanding" is no guarantee that students will develop the related manipulative skills. Instead of the dreams which the high-level mathematicians dream, these are the real facts with which the math teachers in the school system must cope.

Conventional Method of Instruction. Changing the content and goals of the mathematics curriculum is not the only change which is needed in mathematics instruction. Based on the data we have reported from high schools, many students enter high school with arithmetic deficiencies which are not remedied, and high school instruction itself leaves much to be desired. It is fair to say that the conventional lecture-discussion method of mathematics instruction simply does not communicate with average and below-average students. And in spite of the fact that this method of instruction would be more successful if math teachers knew more about the process of learning mathematics, the method has inherent flaws which set too low a ceiling on what it can accomplish. The basic flaw is the fact that attention cannot be given to the individual student so that his unique learning process can be controlled. Any method which does not offer the possibility of personal attention to average and below-average students is a dehumanizing method for them because they cannot learn at anywhere near their maximum potential without personal attention.

Abuse of the Average Math Student.

The absurdity of present math instruction can be seen best by examining what happens to the average student. We will concentrate on his high school training in mathematics, assuming that he takes only the two-year algebra-geometry sequence which is terminal for the majority of students. We can discuss what he does, or does not, know at the end of this two-year period in terms of arithmetic skills, algebraic skills, and geometric skills.

> Arithmetic Skills. He probably has some deficiencies in arithmetic skills when he enters high school. He may be deficient in topics like operations with decimal numbers, fractions, and percents. He may not understand the base-10 number system. He may even be so lacking in number fluency that he cannot add two 2-digit numbers or perform a short division mentally. The high school curriculum makes no organized effort to remedy many of these deficiencies. It also makes no organized effort to increase his numerical fluency or calculation skills. For example, it is improbable that he will either be introduced to the slide rule or shown any formal techniques for estimating answers to calculation problems.
>
> Algebraic Skills. His algebra course will cover many topics which are irrelevant to a student who does not intend to take further math courses. For example, he will study sets, inequalities, division of polynomials, factoring trinomials and handling many contrived fractions with which this type of factoring is necessary, and the real number system. He will be introduced to a great deal of mathematical terminology which has little or no relevance to his needs. Many science-oriented topics will be ignored or virtually ignored. For example, no real emphasis will be given to operations with literal fractions, solving literal equations, or to formula derivations. The coordinate system will probably not be generalized to the graphing of formulas. Empirical graphing of measurements will almost certainly be ignored, and it is likely that the graphing of all non-linear functions will be ignored.
>
> Geometric Skills. The goal of his geometry course will undoubtedly be to develop an insight into Euclid's axiomatic-deductive system. Heavy emphasis will be given to studying the nature of proof and to proving theorems. Because of this emphasis, many fundamental properties and relations of basic geometric figures may well be obscured. The set of applied problems which he attempts to solve may exclude many realistic ones. He may not be introduced to basic trigonometry. Little emphasis will be given to calculation. Little or no emphasis will be given to basic measurement concepts, the rules for calculations with measurements, and basic measurement systems.

The two-year, algebra-geometry sequence for high school students is not designed to be a terminal program. In the total four-year high school sequence, these courses not only serve as a preparation for the more advanced math courses, but they are used as a test to determine which students qualify for the more advanced courses. Though the school system knows that conventional methods of math instruction do not communicate with the average high school student, he is still subjected to this preparation which he will not use and to this test whose outcome is a virtual certainty before the fact. When this two-year sequence is completed, his math skills can probably be summarized by the following statements:

(1) He is probably deficient in arithmetic skills.

(2) He probably cannot use a slide rule.

(3) He probably does not understand measurements and basic measurement concepts.

(4) He probably does not understand the type of algebra which is used in science and technology.

(5) He probably does not understand the type of geometry which is used in science and technology.

After ten years of mathematics instruction in our school system, the school system has left the average student in this position:

(1) He may not even have enough math skills to handle his personal affairs.

(2) He does not have the math skills needed to take anything more demanding than a descriptive science course.

(3) He does not have the math skills needed in the training for a wide range of careers.

(4) Because of the equivalent of a "failure" experience in math courses, his confidence in his ability to learn mathematics is low and his anxiety about mathematics courses is high.

(5) If he wants to take another math course, it clearly must be remedial in nature. Usually the school system offers him only a general math course as an alternative. This general math course usually does not have suitable goals, and it uses a method of instruction which has proven to be unsuccessful in communicating with him.

It is difficult to justify what is done to the average student by math educators. If the present content and the present method of instruction were the only alternatives, we would have to learn to live with them. Since the present content and the present method of instruction are not the only alternatives, we cannot in conscience live with them much longer.

A Minimum Mathematics Curriculum.

If math education is to serve the broader needs of students and our society, its philosophy must be changed. The elementary and high school curriculum must become less oriented to the preparation of professional mathematicians and more oriented to the math skills needed in basic science and technology. In fact, a minimum curriculum should be developed which concentrates heavily on those skills. Insights into the structure of mathematics or the way a mathematician thinks and topics whose sole purpose is a preparation for higher math courses should be delayed until this minimum curriculum is completed. This minimum curriculum should contain the following types of goals:

(1) Numerical fluency and calculation skills, including use of the slide rule.

(2) Algebraic manipulative skills, including emphasis on formula rearrangement and derivations.

(3) Graphing skills with emphasis on the types of functions (straight line, parabola, hyperbola, sine wave, exponential, logarithmic) which are used in science to represent physical phenomena.

(4) Understanding the basic properties of triangles, circles, quadrilaterals, and related solid figures.

(5) Trigonometric skills in solving right and oblique triangles and vector problems.

(6) An understanding of measurement concepts, measurement systems, empirical graphing, variation, formula evaluation, and some rudimentary dimensional analysis.

(7) An introduction to the concept of probability and some basic statistical concepts.

Though the objectives listed above are merely a crude outline, they deviate markedly from the basic objectives of the present math curriculum. If a minimum curriculum of this type were adopted, the main consideration should be student learning and not a mere coverage of content. Therefore, the whole concept of separate courses with a fixed content would have to be abandoned. Though some students might not complete this curriculum before graduating from high school, the faster students would be able to complete it in a much shorter time.

The minimum curriculum which we have outlined would be beneficial for all students. Its benefits for the average or slower students are clear. A higher percentage of them would be capable of taking science or technical courses in high school; a higher percentage of them would graduate from high school with the math skills needed in the training programs for many skilled or technical jobs. Its benefits for the faster students are also clear. They would be prepared to take mathematically-oriented science courses at an earlier age. They would also be better prepared to take higher-level math courses for all of the following reasons:

(1) Manipulative skills are a necessary condition for higher-mathematics instruction, and college math teachers are presently complaining that many high school graduates do not have the manipulative skills which more advanced math courses require.

(2) Learning the structure of axioms and principles would be facilitated by a prior understanding of them in the context of manipulation.

(3) Introducing the basic concepts of trigonometry in the context of solving triangles is a useful foundation for the study of analytical trigonometry.

(4) Numerical fluency and calculation skills, including the use of a slide rule, would eliminate the necessity of avoiding anything other than simple, contrived calculations in math instruction.

(5) Calculation skills would make possible a more numerical and intuitive approach to fundamental principles, thereby increasing the understanding of those principles.

New Method of Instruction.

Changing the content of the mathematics curriculum is not the only change which is needed in mathematics instruction. If we want to guarantee a mastery of fundamental skills for a high percentage of students, a different and more flexible method of instruction will be needed. This method of instruction must be able to cope with individual differences in speed in learning. It must also be able to cope with individual differences in retention. For many students, a well-designed strategy of distributed practice will be necessary. This strategy should include comprehensive exams as well as accompanying formal review and remedial periods. It should be obvious from the data we have reported from conventional high school classes that the current instructional method is not successful with average and below-average students. We suspect that it is also too slow and inefficient with the faster student. Furthermore, the conventional method has the inherent deficiencies of a group method of instruction, and so it cannot provide the individual attention which will be needed if student mastery is taken as a serious goal for all students.

A method analogous to the system of instruction developed by the project staff should be developed for math instruction at both the elementary and high school levels. This system of instruction has demonstrated that average and below-average students can learn mathematics from a combination of programmed materials and tutoring. A system of instruction of this type not only guarantees individual attention for each student, but it is also flexible enough to cope with individual differences in speed in learning and retention.

Education in General

Though less than 40 percent of high school graduates enroll in four-year academic colleges and the number who receive a baccalaureate degree is considerably smaller, the major effort in our educational system is devoted to this minority of students. The needs of the majority of the students are given either token consideration or totally ignored. Few, if any, school systems have a well-conceived track for the students who lack either the ability or interest to enroll in a four-year academic college. If the needs of this majority of students are to be met, mathematics instruction is not the only type of instruction in which changes are needed. Changes in both curricula and methods of instruction are needed on a much broader basis. In this section, we will discuss this need for change under the following headings: (1) need for change, (2) institutional resistance to change, (3) need for systems of instruction, (4) personnel needed to develop and implement systems of instruction, and (5) the cautious use of educational hardware.

Need for Change.

Any astute observer of our educational system realizes that there is something basically wrong with it. Most of the educational money is used to support college-degree programs, or the type of elementary and secondary education which is a preparation for college-degree programs. This type of investment was acceptable in the 1930's when the job market could easily absorb the students who were not college graduates. It will be a disaster in the 1970's since the unskilled job market is diminishing and the need for skilled workers without four-year college degrees is rising rapidly. The educational system has simply not kept pace with the rapidly changing demands of the job market. This fact is reflected in the general lack of vocational and technical training in our secondary schools. Since less than 20% of the jobs in this country require a four-year college-degree, and there is no reason to believe that this percent will change radically, a general reorientation of the goals of our educational system is a necessity.

Besides a general reorientation of goals, it is also obvious that the current methods of instruction are inadequate. Though the public began to realize this fact when the schools proved ineffective with culturally deprived students, the ineffectiveness is much more widespread. The lecture-discussion method, which is firmly entrenched in the schools, has never been successful with more than the top third of students at best. It is not a poor method because it is old; it is a poor method because it is unsuccessful with too many students. This method has demonstrated convincingly that it does not communicate adequately with average and below-average students. Since the need for successful communication with them exists, obviously new and better methods of instruction are needed.

Institutional Resistance to Change.

It would be safe to say that the educational system is one of the most conservative institutions in the United States. Though vast changes in the school system are needed, the changes are not occurring at a very rapid rate. The reason why the schools are so sluggish in making necessary changes is puzzling. Either the current school personnel is too inept to change, or the school culture does not readily permit change, or the changes needed are so vast that nobody knows exactly where to begin. There are various facts about the educational system which actually hinder changes. We have listed several of them below:

(1) The public school system is becoming more and more of a monopoly. Since it has no substantial competition and since few parents have another alternative, it does not have to improve in order to survive.

(2) Adequate assessment is not common. With the philosophy of the self-contained classroom, even school administrators are not aware of the achievement level of the students in an empirical sense. It is difficult to see how an organization which does not precisely measure its current effectiveness can be much concerned about it.

(3) The education profession has no incentive system for performance. Teachers are paid in terms of their own educational training and their number of years of service. Ineffective teachers may well be paid more than effective ones.

(4) The monetary rewards in the teaching profession are too low. Therefore, the profession does not attract enough talented members, and it will not do so until the monetary rewards become more substantial.

(5) Teachers know too little about the learning process. This lack of knowledge is the fault of teacher-training programs.

(6) The attention of school administrators is so absorbed with matters like physical facilities, personnel, and scheduling that they cannot devote much attention to improvements in instruction. Furthermore, the fact that little or no money is budgeted for research, development, experimentation, and assessment indicates that school administrators have not committed themselves to improvement in instruction.

(7) Many innovations, like visual aides, the use of television, and the use of teacher aides, are designed to support the conventional teaching method. The basic adequacy of that method is not challenged.

(8) The physical facilities of schools and their scheduling practices are designed to support the conventional teaching method in a self-contained classroom. Most schools do not have the facilities for a learning center like the one used to teach Technical Mathematics to the technicians at MATC. Furthermore, the use of a learning center requires some changes in scheduling, and in many schools, scheduling is given higher priority than student learning.

As resistant to change as our educational system is, changes in it are inevitable because they are necessary. These changes will either be instigated from within the system, or they will be forced on the system from the outside. Changes are needed in both curriculum and in methods of instruction. Curriculum changes alone, as has happened in many recent national projects, are not enough. Much more emphasis must be given to student learning since curriculum changes are useless if the instructional method does not communicate with the students. Furthermore, curriculum changes should be made in terms of what is relevant to the needs of students rather than what is relevant to the needs of content specialists. For example, "thinking like a mathematician," "thinking like a physicist," or "thinking like a transformational grammarian" are hardly goals which are relevant to many students in our society. And unfortunately, while lofty goals of this type are pursued, many goals which are more relevant to most students are totally ignored.

Need for Systems of Instruction.

Though the system of instruction which has been developed by the project staff is far from complete or perfect, it does suggest the direction which future efforts for innovative changes in education should take. It seems obvious that more systems of instruction of this type must be developed and implemented in schools, with initial efforts given to basic skills like reading, communication skills, mathematics, basic science and basic technology. These systems should include the following features: A content which is relevant to the needs of students, a method of instruction which is flexible and designed to maximize interactions between the teacher and individual students, learning materials which take into account what is known or being discovered about the learning process, refined and continual assessment, and an openness to any changes which are necessary to increase the level of achievement of the students. These systems of instruction can be developed by small staffs, provided that the staff members are able and well-trained. As the systems are developed, they can be made generally available to the educational community. If our society wants to get the greatest educational gains for the money it invests in educational improvements, the money should clearly be invested in model systems of instruction of this type.

Personnel Needed to Develop and Implement Systems of Instruction.

During the course of the mathematics project, the specific types of personnel needed to develop and implement a system of instruction became clearer. The necessary personnel can be divided into various categories: instructional-materials experts, operations experts, and various types of paraprofessional or supportive personnel. The qualifications needed for these various types of personnel will be discussed in the following paragraphs.

Instructional-materials experts are needed for the development of learning materials and assessment instruments. This group must include experts in content, test-construction and the production of learning materials. The content expert should be able to judge learning objectives in terms of their relevance for students. He must clearly understand the difference between the objectives for terminal and non-terminal courses.

The test-construction expert must be able to design test items which adequately assess the learning objectives. He must understand the learning process well enough so that he recognizes the difference between transfer and non-transfer items. The learning materials writer must be well-informed about the learning process and the learning characteristics of the students. He must also be creative enough to communicate the content to the students. Though it is possible that one person could fill all three roles, it would probably be better to have a group so that interactions would be possible. However, the group should remain small so that progress is not hampered by excessive interactions.

Operations experts are needed for the general implementation of the instruction. They must be experts in controlling the students' motivation and learning processes, and in developing a classroom procedure which maximimally supports both of these types of control. In order to control student motivation, they should be knowledgable about human motivation. Perhaps they should even be trained in some sort of behavior therapy. In order to control the learning processes of students, they must be experts in tutoring. Since they will be assisted by teacher aides in the latter function, they must be able to train teacher aides in the art of tutoring. The operations experts must also be able to offer constructive criticism to the learning-materials experts so that their experience can be incorporated in revisions of the instructional materials.

Two types of paraprofessional or supportive personnel are needed. The two types are teacher aides and secretarial help. Teacher aides are needed to perform functions like taking attendance, test-correction, and tutoring. In order to do tutoring, they must be familiar with the learning materials, and they must be trained in tutoring techniques. Because their functions are limited, they do not need a four-year college degree. The teacher aides could either be a special group of non-students, or they could be students who have already completed the course. Secretarial help is needed to keep student records and to organize the data for analysis. They must be familiar with simple statistical concepts like mean, median, distribution of scores, and item analyses.

If systems of instruction are going to be developed and grow, some mechanism will be needed to train the types of personnel described above. Teacher aides and secretarial help should be the easiest to train. The operations experts can probably be recruited from among the ranks of the ordinary classroom teacher. However, it is highly doubtful that many of the learning-materials experts can be recruited from the ranks of the ordinary classroom teacher. Many classroom teachers are not interested in developing systems of instruction, and those who are interested are not trained to do so. They usually do not have a sophisticated understanding of the learning process, and they are not highly trained in such skills as test-construction or the writing of learning materials. It is also questionable whether the ordinary classroom teacher, even with the proper training, has the ability to develop successful systems of instruction. The best alternative at the present time is to recruit learning-materials experts from existing professionals, and to initiate programs to train more professionals of this type.

The underlying goal of any system of instruction should be to produce as much learning as efficiently as possible. In order to achieve and maintain this goal, some incentive program should be offered to the personnel, especially the learning-materials experts and the operations experts. An incentive should be offered to the operations experts for two reasons: (1) to reward them for developing a classroom procedure which is effective, and (2) to encourage them to reduce the cost of the classroom procedure when such a reduction does not damage the achievement level of the students. An incentive should be offered to the learning-materials experts for two reasons: (1) to maintain a constant effort from them, and (2) to encourage talented people to undertake this type of challenging and difficult work. An incentive program of this type would fill a void in the current educational system in which there is little relationship between effectiveness and monetary rewards. It should be possible to develop systems of instruction with an incentive factor whose total cost is less than the cost of traditional educational methods but whose effectiveness is much greater than traditional educational methods.

A Cautious Use of Educational Hardware.

In recent years, there has been an increasing interest in the use of educational hardware like teaching machines, computers, film strips, film loops, video tapes, and audio tapes. Though the intensity of the early enthusiasm for the use of hardware has somewhat waned, it is easy to understand the intense enthusiasm of those who advocated its use in some form. They undoubtedly felt that the use of hardware would enable the school system to make a quick quantum jump in its instructional effectiveness. Underlying this hope was a somewhat naive belief that hardware per se had some magical properties over and above the quality of the educational materials or software. Most people are now willing to admit that this belief was false.

In order to get educational hardware in perspective, we must remember what the essence of the educational process is. With student learning as its goal, the essence of the educational process is communication. The success of any instruction is based on the quality of the communication. Teachers, printed materials, and hardware are merely "media" or "means" by which this communication can be accomplished. There are no inherent properties in any of these "media" which can compensate for ineffective communication. For example, if a certain way of presenting the concept of a logarithm does not communicate with students, it is irrelevant whether it is presented in written materials, presented orally by a live teacher, presented by a teacher on video tape, or presented in some form by a computer-based system of instruction. The inability to communicate effectively is the real problem which education faces. This problem will not be resolved until the educational community develops a more sophisticated knowledge of the learning process. Until that time, no educational medium can approach its inherent potential.

When systems of instruction are developed, the initial efforts must be concentrated on the development of effective learning materials or software. Without such materials, the goal of student learning will not be attained. However, any system of instruction should be open to the use of educational hardware. For example, it seems obvious that film loops or film strips would be extremely useful in science instruction. It also seems obvious that a computer-based learning system would offer a unique capability of controlling the learning process in mathematics instruction. However, the use of hardware should be dictated by its demonstrated effectiveness in offering some unique capability which other, less expensive, media cannot offer. The use of hardware for its own sake is ridiculous.

When discussing the use of hardware in systems of instruction, the question of cost has to be seriously considered. It is common knowledge that hardware and the development of materials for hardware are expensive. Though our society can and should support experimentation in education with all types of media, it should do so with the understanding that some of the products of this experimentation will be beyond the budgets of most school systems at the present time. Even though it is very effective, a system of instruction will not be widely adopted if it is high-priced. The only systems of instruction which stand a chance of widespread adoption at the present time are those whose cost is minimal.

Future Directions

What has been accomplished by the project up to this point is a mere beginning. The project staff naturally has many ideas about directions which the project can and should take in the future. These future directions will be discussed in this section. They will be discussed under the following headings:

(1) Promoting a more widespread use of the system of mathematics instruction which has been developed.

(2) Further development of the system of mathematics instruction for technical schools and high schools.

(3) Development of systems of instruction for physics and basic technical courses.

(4) Experimentation with the use of computer-based systems of instruction.

(5) Some basic research which is needed to improve instruction.

(6) Development of a national center for research on curriculum development and instruction for technical and vocational education.

(1) Promoting a More Widespread Use of the System of Mathematics Instruction.

During the course of the project, the project staff has been somewhat hesitant to publicize its findings. In some ways, this hesitancy was necessary because the staff was very small and the full efforts of each staff member were needed to develop the system. However, the hesitancy would have occurred anyway since the staff members shared a philosophy that reports should not be issued until the effectiveness of the system could be supported with empirical data. Now that the results are being published on a more widespread basis, it seems logical that the system of instruction should be made available on a national basis because it fills a national need. In order to do so, the following steps should be taken:

(1) The instructional materials should be commercially published.

(2) Institutes should be offered to train teachers in the use of the materials and the system of instruction.

The need to make the materials commercially available seems obvious in view of the national need for better and more relevant mathematics instruction for average and below-average students. The need for teacher institutes seems just as obvious. Not only could these institutes be used to promote the use of the system of instruction, but they could serve as a means of acquainting teachers with the general method of instruction, the criterion for selecting content, the learning principles which are incorporated in the programmed materials, and efficient methods of tutoring. These institutes, which should be run in conjunction with the actual teaching of a group of students, should be offered to math teachers in both vocational-technical colleges and high schools. One goal of these institutes would be the development of a math program with continuity from high schools to vocational-technical colleges. Though eventually offered on a national basis, perhaps the first institute should concentrate on the development of such a cooperative program between the Milwaukee Area Technical College and local high schools. The product of this cooperative development could then serve as an object lesson for other areas in the United States.

(2) Further Development of the System of Mathematics Instruction.

During the four-year history of the project, the staff members have developed a very broad view of mathematics instruction. This view is based on the national need for a more science-oriented content and a more successful method of instruction, especially for average and below-average students. It encompasses the needs of students in vocational-technical colleges, high schools, the possibly elementary schools. Because of the relevance of its content and its effectiveness, the system of math instruction which has been described in this report would play a major role in this broad view of mathematics instruction. However, the content covered would have to be expanded, and the materials would probably have to be written in various ways to communicate more successfully with students of various ages, math skills, and learning abilities.

In this section, we will discuss various ways in which the present system of math instruction could be expanded and modified to fill a much broader need. The discussion will be divided into three sections: (1) mathematics for technicians, (2) mathematics for other students in vocational-technical colleges, and (3) mathematics for high school students. When other courses for college-age students or courses for high school students are discussed, it should be remembered that the materials developed for technicians or adaptations of these materials can serve as the core materials for many courses.

Mathematics for Technicians. Many of the booklets for the Technical Mathematics course need revision. This revision is especially needed with the booklets which cover more advanced topics. In addition to the revision of existing booklets, additional booklets are needed to complete the coverage of math content required in technical training. In this section, we will list the major revisions and additions needed for the Technical Mathematics course itself, and the new materials needed for two additional courses: "The Quantitative Aspects of Science" and "Technical Calculus."

Technical Mathematics. The following major revisions or additions are needed for the Technical Mathematics course:

(1) The calculation booklets should be revised, and additional calculation topics should be covered.

(2) Some additional topics in algebra, especially in the area of derivations, are needed.

(3) The geometry booklets should be better organized, and additional topics in geometry should be added for the students in the civil and mechanical technologies.

(4) Booklets covering determinants and sine-wave analysis are needed for the students in electrical technology.

Quantitative Aspects of Science. Booklets covering the quantitative aspects of science are needed. The following topics should be covered:

Basic Measurement Concepts
Measurement Systems
Rudimentary Dimensional Analysis
Formula Evaluation
Empirical Graphing
Variation

Since additional booklets cannot be included in the Technical Mathematics course, a new course should be developed for all entering technical students. The calculation booklets, which are now covered in the Technical Mathematics course, should be included at the beginning of this new course. Since an introduction to some of the basic concepts of physics would be included in this new course, it could serve as a preparation for the physics course which all technical students take.

Technical Calculus. Though not all technical students are required to take a calculus course, materials should be prepared to teach basic calculus to those students who need such a course.

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Emphasis should be placed on the calculus topics which are most useful in elementary science and technology, including an introduction to differential equations. An attempt should be made to introduce these topics in as intuitive a manner as possible, with a corresponding deemphasis of proofs and derivations.

Mathematics for Other Students in Vocational-Technical Colleges. Math materials are needed for other students besides technicians in vocational-technical colleges. In order to serve the needs of these other students, the following sets of materials should be prepared:

Basic Arithmetic and Number Fluency. Materials should be developed to teach basic arithmetic operations and number fluency. The present materials for the Technical Mathematics course presuppose these skills. However, many entering students in the technical programs and other programs are deficient in them. Effective materials of this type would have a widespread use in our society.

Apprentices and Skilled Tradesmen. Though some of the Technical Mathematics materials in their present form can be used in courses for apprentices and skilled tradesmen, they are not ideally suited for them. A set of materials should be developed specifically for these courses. These materials would include some basic algebra, basic geometry, basic trigonometry, and slide-rule skills. They should also include a review of arithmetic operations.

Intermediate Algebra. A special course in intermediate algebra should be developed for those vocational-technical colleges which include an academic junior college. This course, which would be remedial in nature, is needed for many students as a preparation for a regular "College Algebra" course.

Mathematics for High School Students. It should be obvious that any math materials developed for the various programs in vocational-technical colleges would be extremely useful for high school students. In fact, since these materials would prepare high school students for elementary science and technical courses, and ultimately for vocational and technical careers, they would be more relevant than the present high school content for most high school students. The materials could be used in high schools in either of the following two capacities:

Technical Mathematics Course. A Technical Mathematics course should be developed to replace the present "General Mathematics" high school course for average and below-average students. If offered as a two-year program, each student could proceed at his own pace through the materials prepared for technician-training in vocational-technical colleges. Some students might not complete the entire set of materials, and others might have to begin with the materials prepared to teach basic arithmetic operations. Besides preparing students for vocational and technical careers, this course could serve as a vehicle for recruiting students into the training programs for such careers.

> Minimum Mathematics Curriculum. If the minimum mathematics curriculum for elementary and high schools which we described earlier were adopted, the materials from the Technical Mathematics course could serve as the terminal materials in this curriculum. If the materials were used in this capacity, all entering high school students would be required to complete them as an entrance requirement for more advanced math courses.

Considering the fact that much math instruction in vocational-technical colleges is remedial in nature, it might be more efficient at the present time to concentrate efforts on systems of math instruction for high school students. Not only is more time available in high school, but the resulting higher entry skills in students enrolling in vocational-technical colleges would permit the development of broader math skills in the math courses in those colleges. If the major effort were devoted to high school courses, the materials should be revised in order to achieve the most successful instruction of high school students. This revision would have to include a coordination with math instruction in elementary schools and junior high schools, and it might have to include a development of similar systems of instruction for those particular schools.

(3) Development of Systems of Instruction for Physics and Basic Technical Courses.

Just as a system of instruction was needed to communicate mathematics effectively to technical students, systems of instruction are needed to communicate the basic concepts and principles of physics and basic technology to these same students. The latter systems of instruction are needed because the conventional courses in physics and basic technology suffer from the same deficiencies which are encountered in conventional mathematics instruction. That is, the content is not always relevant enough, and the method of instruction is not as effective as it could be. Besides a Technical Physics course, systems of instruction should be developed to teach the basic principles of electronics, combustion engines, hydraulics, strength of materials, and so on. Just as the materials developed for the Technical Mathematics course are useful in high schools, the materials developed for these courses would also be useful in a vocational-technical track in high schools.

The initial effort should probably be devoted to a Technical Physics course. Since the present achievement level in this course is quite low, some alternative to the traditional lecture-laboratory method of instruction must be developed. Furthermore, the content and goals of the present course should be reexamined, since they seem to be more oriented towards "physics for physicists" than towards "physics for technicians." A "physics for technicians" course should include an emphasis on understanding the principles of physics which are relevant in technical work plus the ability to apply these principles in elementary problem-solving. Besides improving the instruction in the Technical Physics course itself, the development of an effective system of instruction for that course would serve the following three useful purposes:

(1) The Technical Mathematics and Technical Physics courses could be blended into an integrated package in which each supports the other. Therefore, the Technical Physics course would make the Technical Mathematics course come alive in the sense that students would be able to use their math skills immediately in a meaningful context.

(2) The Technical Science course could be used as a vehicle for teaching formal strategies for problem-solving. Strategies could be taught which would generalize to problem-solving in other technical courses. These strategies cannot be taught in the math course itself since they presuppose an understanding of physical or technical concepts and principles.

(3) Any method of instruction which is effective and efficient in communicating the content of a physics course should be effective and efficient in communicating the content of any basic technical course.

(4) Computer-Based Systems of Instruction.

As basic instructional materials are developed, a concurrent attempt should be made to develop computer-based systems of instruction which utilize these materials. Though the cost of developing computer-based systems of instruction is very high, and though the cost of using computers in education is now too great for them to be of general use, adapting materials to a computer-based system would have two advantages. First, a more precise frame-by-frame assessment of the learning materials would be possible, and therefore their quality could be substantially improved. And second, systems of instruction could be prepared in anticipation of the day when the cost of computer-based education will be low enough to permit its more widespread use in schools or homes. Computer-based systems will eventually be involved in all types of skill development because it is difficult to conceive of a more efficient mechanism for the development of skills. Only a computer can reasonably administer the one-to-one, individual practice which is necessary in the development of a skill. For example, the whole level of mathematics education could be greatly upgraded if a computer system for developing arithmetic skills and number fluency were available in elementary schools and high schools. At the present time, the educational community has virtually no means of administering controlled individual practice of this type.

(5) Basic Research Needed to Improve Instruction.

As the project has developed, the need for various types of basic experiments in the learning process has become clear. The experimental questions flow from problems which have been encountered in actual instruction, and answers to them are needed in order that instruction can become more effective. Like most significant educational questions, they cannot be answered within the context of a one-hour or relatively brief experiment involving artificial tasks. But even though considerable effort would be needed to do this experimentation, the experiments would contribute to a basic understanding of the process of classroom learning, and the findings could be put to great use by personnel who prepare instructional materials. Therefore, the long-range gains for educational effectiveness would be substantial. A few of the possible experimental questions in the area of math instruction are outlined below.

Role of Verbal Language in Mathematics Learning. Having done a complete reversal during the four years of the project, members of the project staff now hypothesize that the ability to describe mathematical processes and procedures in words is extremely beneficial to the student. By "describing in words," we do not mean the precise statements of a professional mathematician. We mean statements in the student's own words which include some technical terms and generally have the same substantive meaning as the statement of a professional mathematician. This "ability to describe in words" should be examined in terms of its effects on learning, transfer, and retention. It could probably be investigated most easily in the context of solving simple equations.

Though there is a need to examine the effect of forced "verbalization" for all students, this information would be especially useful in the design of instruction for the very slow learner, including the culturally disadvantaged. It seems that many students of the latter type do not have the habit of translating what they are doing with mathematical stimuli into words. Some educators are suggesting that the culturally deprived learn in a non-abstract, somewhat rote manner, and that non-verbal instruction should be designed to fit their current manner of learning. Members of the project staff feel that the opposite tack is probably better because "thinking" is highly related to the ability to use abstract verbal language. Therefore, staff members advocate a type of math instruction for the culturally deprived in which one of the terminal goals is the ability to describe verbally various mathematical processes and procedures. They hope that this emphasis in instruction will overcome some of the obvious deficiencies in the learning process of such students.

Transfer in Mathematics Learning. Ordinary mathematics textbooks and mathematics instruction seem to assume a fair amount of transfer. However, during the course of the project, staff members have found that the amount of transfer which occurs in many students is negligible. Though many more subtle instances of a lack of transfer occur, here are a few examples of obvious ones:

(a) One-letter equations, two-letter functional relationships with numerical coefficients, and formulas or literal equations form three distinct sets of stimuli for many students. For such students, manipulative skills with one set do not automatically transfer to the other two sets.

(b) Definitions or procedures learned with the exclusive use of standard-position figures in geometry do not automatically transfer to figures in non-standard positions. This is true for the definition of the sine, cosine, and tangent of an angle. It is also true for the formula for finding the area of a triangle, and the use of the law of sines and the law of cosines.

Some basic experiments are needed to determine the exact amount of transfer which occurs from one type of stimulus to another. These empirical facts are needed so that the instructional materials can be designed to cope with the transfer problem. Furthermore, some basic experiments are needed to determine how transfer can be most efficiently accomplished, and under what conditions it can be improved.

Retention in Mathematics Learning. The major effort in the project up to this point has been focused on student learning. Though always interested in the question of retention, staff members felt that investigations of retention would be premature if rather high learning levels could not be initially attained. During the past year, some preliminary probes were made into the area of retention by means of comprehensive exams. Empirical studies are needed of even longer range retention. Besides empirical studies of the retention rate, techniques must be developed to cope with forgetting and to guarantee as high a level of long-range retention as possible. It might be possible to decrease the amount of forgetting by a greater emphasis on the use of verbal language in the original learning. And some type of distributed practice, either in the form of review items during daily tests or concentrated periods of review, will be necessary to counteract the forgetting which will certainly occur. If concentrated review periods are used, some experimentation will be needed to determine whether one of the many possible methods for this type of review is the most efficient and effective.

(6) National Center for Research on Vocational and Technical Education.

In this chapter we have outlined various types of systems of instruction which are needed in the areas of mathematics, basic science, and basic technology in order to improve vocational and technical education at both the college and high school levels. Even without including other major areas of instruction, like reading and communication skills, it should be obvious that a small staff will be unable to develop all of the systems of instruction which we have proposed. The development of these systems is a time-consuming and arduous process, and the perfection of each system will require long and concentrated efforts if the greatest amount of student learning is the expressed goal. Furthermore, since the systems will assume responsibility for communicating rather complex content to students with whom traditional methods of instruction have not been highly successful, the personnel developing them will have to be creative enough to develop new and more successful methods of instruction.

In order to facilitate the development of the many systems of instruction which are necessary, a national center should be established for research on curriculum development and instruction for vocational and technical education. In a center of this type, attention could be focused on the major problems of educating students for vocational and technical careers. Though one small group of staff members would concentrate exclusively on the continuing development of a single system of instruction, the various small groups could interact and share ideas. As soon as each system of instruction passed an experimental stage, it would be made available on a national basis. Since the center would be a national one, the priorities of attention would be dictated by national needs rather than local needs. Furthermore, the personnel could be recruited on a national rather than a local basis. Since the personnel would be accepting a challenge to produce learning in students with whom our educational system has had minimal success, national recruiting would be necessary to find personnel with the training and creativity to be significantly successful.

Rather than being situated on a university campus, it would be better if this national center were situated at a large vocational-technical college like the Milwaukee Area Technical College. There are many reasons why it would be more successful if it were situated on a vocational-technical college campus. Staff members would have access to students in an actual school setting, and since they would be faced daily with the problems of communicating with average and below-average students, their goals and efforts would be much more realistic. Furthermore, the staff members should be responsible for the actual instruction of students because the deadlines of classes which must be taught would accelerate their efforts. And since the systems of instruction would have to be designed to fit into an ongoing school operation, there would be a guarantee that the finished products would actually be useable in schools. In order to free the staff members from involvement in too many local issues, a center of this type should probably be administratively autonomous from the local school.

In view of the great need for improvement in the instruction of average and below-average students and the limited funds available for educational improvements in the United States at the present time, the funding of a national center of this type would probably be the wisest investment that could currently be made. Instead of diffusely spending money in various local areas where trained talent might not be available, funds would be concentrated on intense efforts to make major breakthroughs in communicating significant subject matter to students who have been long neglected in our school system. The present report suggests that major breakthroughs of this type are possible. If a staff of highly talented and highly trained personnel were concentrated in one center, hopefully the rate of these major breakthroughs could be accelerated.

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APPENDIX A

CHARACTERISTICS OF THE ENTERING TECHNICAL STUDENT MILWAUKEE AREA TECHNICAL COLLEGE (1968-69)

CHARACTERISTICS OF THE ENTERING TECHNICAL STUDENT
MILWAUKEE AREA TECHNICAL COLLEGE (1968-69)

Information about the general characteristics of the entering technical students is available from three sources: (1) a personal data sheet filled out at the beginning of the Technical Mathematics course, (2) high school transcripts, and (3) ACT scores. The purpose of this report is to summarize information available from these three sources for the 480 students entering in September, 1968.

Personal Data Sheets. From the personal data sheets, we can report these pieces of information: (1) technology entered, (2) age distribution, (3) previous attendance at college, (4) high school or college math courses, (5) ability to operate a slide rule, and (6) completion of a high school physics course.

Because a few of the personal data sheets were not completely filled out, there is some slight discrepancy in the number of students included in each summary. This discrepancy is too slight to affect the overall picture.

In the table below, the number of students enrolling in each technology and the percentage of students enrolling in each technology are given:

	N	%
Air Cond. & Refrigeration	13	2.7
Chemical	10	2.1
Civil	106	22.1
Combustion Engine	24	5.0
Dental	19	4.0
Electrical	149	31.0
Fire	1	0.2
Fluid Power	11	2.3
Mechanical	59	12.3
Metallurgical	8	1.7
Photo - Instrumentation	12	2.5
Printing & Publishing	34	7.0
Junior College	26	5.4
No Technology Listed	8	1.7
Total	480	100.0

The age distribution is given in the following table:

AGE	N	%
24-43	30	6.3
23	14	2.9
22	25	5.2
21	28	5.9
20	38	8.0
19	101	21.2
18	201	42.1
17	40	8.4
Total	477	100.0

Though the age of the students ranges from 17 to 43, 72% of the students were either 17, 18, or 19 when the course began. This distribution suggests that the majority of the students are entering the program immediately after their high school graduation. Only 96 students (20%) reported that they had previously attended some type of college.

High school math courses can be divided into college-preparatory (algebra, geometry, trigonometry, etc.) and non-college-preparatory (general math). The prerequisite for Technical Mathematics 1 is one year each of algebra and geometry, although this prerequisite is occasionally waived. Some students took more college-preparatory courses than these two; others took a General Math course. Of the 96 students who had previously attended college, 81 took some type of college math course. In the table below, we have categorized the number of students who took various types of high-school math programs. For each category, the number of students who took further math in college is given in parentheses.

	N	%
Algebra and Geometry	152 (21)	31.9
Algebra, Geometry, General Math	79 (8)	16.6
Algebra, Geometry, plus at least one more college-preparatory course	221 (52)	46.3
Less than two years of college-preparatory (with or without General Math)	25 (0)	5.2
Total	477 (81)	100.0

Only 46% of the students took more than the two required college-preparatory math courses. Though the 81 students who had taken some type of college math course were asked to specify both the course and the grade received, some did not do so. The reported grades were typically average or lower. Some of the students had taken only a technically oriented math course.

34% of the students claimed some ability to manipulate a slide rule; 34% reported taking a high school physics course.

__High School Transcripts.__ The second source of information about the technical student is his high school transcript. These transcripts were available for only 387 students. Of the 93 students whose transcripts were unavailable, 38 had previously attended some type of college. From these transcripts, we can report (1) the high school rank of the students, and (2) the grades received in the first two semesters of algebra and geometry in high school.

Since the high school rank was not given on 73 transcripts, the ranks of only 314 students were available. In percentiles, the available ranks ranged from 1 to 99. In the table on the next page, the ranks are given in deciles.

Rank in Deciles	No. of Students	Percent
91 - 100	5	2%
81 - 90	8	2%
71 - 80	12	4%
61 - 70	21	7%
51 - 60	41	13%
41 - 50	37	12%
31 - 40	47	15%
21 - 30	53	17%
11 - 20	51	16%
0 - 10	39	12%
Total	314	100%

The high school ranks range from very low to very high. 72% of the students were in the bottom half of their high school classes and 37% of the students were in the bottom quarter of their high school classes. The median rank was at the 32nd percentile.

Though two semesters of high school algebra and two semesters of high school geometry are a prerequisite for Technical Mathematics 1, in special cases students are allowed to enroll who have not completed this entire sequence. In the table below, the grades for this four-semester sequence of courses are given for the 387 students for whom transcripts were available. Since some students did not have a grade listed for one or more of these courses, a "not taken" category was included.

	ALGEBRA I				GEOMETRY I			
	Semester 1		Semester 2		Semester 1		Semester 2	
	N	%	N	%	N	%	N	%
A	15	4%	18	5%	16	4%	12	3%
B	72	18%	65	17%	68	18%	60	16%
C	154	40%	133	34%	123	32%	126	33%
D	130	34%	149	38%	138	36%	138	36%
U	3	1%	7	2%	8	2%	7	2%
Not taken	13	3%	15	4%	34	9%	44	11%
	387		387		387		387	

The percent of students receiving either a "D" or "U" or "not taking" a course increased from 38% in the first semester of algebra to 49% for the second semester of geometry. Similarly, the percent earning an "A" or "B" decreased from 22% in the first semester of algebra to 19% in the second semester of geometry. The overall picture does not represent a high level of achievement in the two prerequisite math courses, especially since the criterion for a "C" or "D" is not clear.

ACT Scores. The third source of information about the technical student was scores from the American College Testing program. These scores were available for only 268 students. In the table on the next page, the average scores for these 268 students are compared with the average scores of students entering four levels of college institutions. The four levels are: Level 1 (junior colleges and technical institutions offering two but less than four years of college work), Level II (four-year colleges which offer no advanced degrees), Level III (colleges which offer a master's degree but no doctorates), Level IV (colleges which offer both master's degrees and doctorates). The data for these four levels of institutions were reported in the summer of 1966, and so they characterize students who entered the colleges in the fall of 1965.

ACT Mean Scores

Colleges	Eng	Math	Soc Sci	Nat Sci	Comp	Number of Students
MATC (technicians)	16.58	19.17	19.36	20.76	19.12	268
Level I	17.34	17.58	18.89	19.00	18.33	55,482
Level II	18.71	19.19	20.32	20.19	19.73	49,959
Level III	19.48	19.73	21.01	20.85	20.39	70,405
Level IV	20.55	21.88	22.72	22.56	22.05	83,161

Technicians fall in the Level I category (junior colleges and technical institutions). When compared with other students at this level, the technicians at MATC have higher scores in all categories except English. However, with the exception of their natural science scores, their scores are not higher than those of students entering four-year institutions.

Summary. The typical incoming technical student is between the ages of 17 and 19 with no training beyond high school. His high school rank was below average. He has not taken more than two years of college-preparatory math in high school, and though his grades in these courses were originally only slightly below average, they became progressively worse. He has no experience with a slide rule, and he has not taken a high school physics course. Though his ACT scores, except in English, compare favorably with the norms for junior colleges and technical institutes, except for natural science they do not compare favorably with the norms for four-year colleges. In general, he is below average in ability when compared to other college-bound high school graduates, and he has not been a strong academic student.

APPENDIX B

DATA FOR ARITHMETIC PRE-TEST

B-1	Copy of Pre-Test: Arithmetic
B-2	Distribution of Scores for Arithmetic Pre-Test MATC Technical Mathematics Students (September, 1968)
	Item Analysis for Arithmetic Pre-Test MATC Technical Mathematics Students (September, 1968)
B-3	Distribution of Scores for Arithmetic Comprehensive Exam MATC Technical Mathematics Students (April, 1969)
	Item Analysis for Arithmetic Comprehensive Exam MATC Technical Mathematics Students (April, 1969)
B-4	Distribution of Scores for Arithmetic Pre-Test Pius XI High School - Technical Mathematics (September, 1968) (Juniors and Seniors)
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B-5	Distribution of Scores for Arithmetic Pre-Test Pius XI High School Entering Freshmen (September, 1968)
	Item Analysis (Overall) for Arithmetic Pre-Test Pius XI High School Entering Freshmen (September, 1968)
B-6	Distribution of Scores for Arithmetic Pre-Test West Division High School - Technical Mathematics (September, 1968) (Sophomores, Juniors, & Seniors)
	Item Analysis for Arithmetic Pre-Test West Division High School - Technical Mathematics (September, 1968) (Sophomores, Juniors, & Seniors)
B-7	Distribution of Scores for Arithmetic Pre-Test Administered Two Different Times West Division High School - Technical Mathematics (1968-69) (Sophomores, Juniors, & Seniors)
	Item Analysis for Arithmetic Pre-Test Administered Two Different Times West Division High School - Technical Mathematics (1968-69) (Sophomores, Juniors, & Seniors)

PRE-TEST: ARITHMETIC

Directions: Work each problem in the space provided. Show all work. Write each answer in the answer box at the right.

Part I: Whole Numbers

1. 59 + 6 + 287 = ?	2. 2,314 - 795 = ?	1. 2.
3. 39 x 694 = ?	4. 4,654 ÷ 26 = ?	3. 4.

Part II: Decimals

5. 39.7 + 0.085 + 5.64 = ?	6. 2.93 - 0.0836 = ?	5. 6.
7. 6.28 x 0.035 = ?	8. 0.008 ÷ 0.40 = ?	7. 8.

Part III: Percents

9. Express the decimal number 0.085 as a percent.	10. Express 47.3% as a decimal number.	9. % 10.
11. Express the fraction $\frac{1}{4}$ as a percent.	12. Find 15% of 60.	11. % 12.

13. 20 is what percent of 50?

14. 16 is 4% of what number?

13. %

14.

Part IV: Number System

Write each of the following as a regular number:

15. three million twenty-eight thousand seventy-two

16. two-hundred and fifteen thousandths

17. What digit contributes most to the value of 387,194?

18. What digit contributes least to the value of 387,194?

In 5,862.497

19. What digit lies in the hundredths place?

20. What digit lies in the hundreds place?

Complete:

21. 500 ten-thousandths = _____ hundredths

22. 7 tenths = _____ thousandths

23. Round 0.081473 to the nearest thousandth.

24. Round 62,500 to two digits.

15.

16.

17.

18.

19.

20.

21.

22.

23.

24.

Part V: Number Sense

25. Which number is largest?

0.000179 0.00130 0.000927 0.001183 0.000998

26. Is 0.094 or 0.103 closer to 0.099?

27. Is 0.001 or 0.01 closer to 0.00572?

28. What number lies exactly halfway between 8.59 and 8.60?

25.

26.

27.

28.

Part VI: Fractions

Note: Answers which are fractions must be written in lowest terms.

29. Write $\frac{5}{8}$ as a decimal number.

30. Write 0.07 as a fraction.

29.

30.

31. $\frac{3}{5} + \frac{1}{2} = ?$

32. $\frac{5}{6} + \frac{4}{15} = ?$

31.

32.

33. $\frac{11}{16} - \frac{5}{8} = ?$

34. $5\frac{1}{3} - 3\frac{1}{2} = ?$

33.

34.

35. $\frac{3}{5} \times \frac{1}{2} = ?$

36. $\frac{3}{4} \times \frac{5}{6} = ?$

35.

36.

37. $3 \times \frac{5}{8} = ?$

38. $2\frac{4}{5} \div \frac{1}{2} = ?$

37.

38.

39. $\dfrac{\frac{3}{7}}{\frac{5}{8}} = ?$

40. $\dfrac{6}{\frac{2}{3}} = ?$

39.

40.

41. $\dfrac{\frac{5}{6}}{3} = ?$

42. Reduce $\frac{12}{30}$ to lowest terms.

41.

42.

43. Convert this division to a multiplication: $\frac{3}{5}$

44. Factor $\frac{7}{10}$ into two fractions.

43.

44.

45. Factor $\frac{3}{7}$ into a whole number and a fraction.

46. $\dfrac{3\left(\frac{1}{5}\right)}{5} = ?$

45.

46.

47. $\dfrac{\frac{3}{4} + 1}{\frac{3}{4}} = ?$

48. $\dfrac{2}{2 + \frac{1}{2}} = ?$

47.

48.

49. $\dfrac{3\left(\frac{3}{2}\right) - 1}{3} = ?$

50. $\dfrac{1}{2\left(\frac{5}{2} - 1\right)} = ?$

49.

50.

DISTRIBUTION OF SCORES FOR ARITHMETIC PRE-TEST
MATC TECHNICAL MATHEMATICS STUDENTS (SEPTEMBER, 1968)

Mean = 64.0%
Median = 66.0%
N = 475

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
50			
49	2	0.4%	
48	5	1.0%	1.4%
47	8	1.7%	3.1%
46	7	1.5%	4.6%
45	13	2.7%	7.3%
44	13	2.7%	10.0%
43	19	4.0%	14.0%
42	20	4.2%	18.2%
41	17	3.6%	21.8%
40	19	4.0%	25.8%
39	21	4.4%	30.2%
38	17	3.6%	33.8%
37	17	3.6%	37.4%
36	11	2.3%	39.7%
35	21	4.4%	44.1%
34	14	3.0%	47.1%
33	18	3.8%	50.9%
32	14	3.0%	53.9%
31	20	4.2%	58.1%
30	15	3.2%	61.3%
29	13	2.7%	64.0%
28	18	3.8%	67.8%
27	17	3.6%	71.4%
26	17	3.6%	75.0%
25	15	3.2%	78.2%
24	14	3.0%	81.2%
23	4	0.8%	82.0%
22	5	1.1%	83.1%
21	14	3.0%	86.1%
20	11	2.3%	88.4%
19	13	2.7%	91.1%
18	9	1.9%	93.0%
17	9	1.9%	94.9%
16	3	0.6%	95.5%
15	4	0.8%	96.3%
14	6	1.3%	97.6%
13	2	0.4%	98.0%
12	2	0.4%	98.4%
11	2	0.4%	98.8%
10	1	0.2%	99.0%
9	2	0.4%	99.4%
8			
7			
6	1	0.2%	99.6%
5	1	0.2%	99.8%
4	1	0.2%	100.0%
	N = 475	100.0%	

ITEM ANALYSIS FOR ARITHMETIC PRE-TEST
MATC TECHNICAL MATHEMATICS STUDENTS (SEPTEMBER, 1968)

Mean = 64.0%
Median = 66.0%
N = 475

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Correct	Incorrect	Not Attempted	Topic	Item No.	Correct	Incorrect	Not Attempted
Whole Numbers	1.	90%	10%		Number Sense	25.	83%	16%	1%
	2.	91%	9%			26.	77%	23%	
	3.	75%	25%			27.	54%	46%	
	4.	81%	18%	1%		28.	84%	12%	4%
Decimals	5.	86%	14%		Fractions	29.	60%	31%	9%
	6.	79%	20%	1%		30.	84%	13%	3%
	7.	75%	24%	1%		31.	86%	13%	1%
	8.	65%	32%	3%		32.	62%	35%	3%
						33.	90%	8%	2%
Percents	9.	73%	23%	4%		34.	75%	22%	3%
	10.	72%	22%	6%		35.	72%	26%	2%
	11.	92%	8%			36.	55%	43%	2%
	12.	72%	23%	5%		37.	76%	20%	4%
	13.	71%	23%	6%		38.	53%	38%	9%
	14.	55%	33%	12%		39.	53%	27%	20%
						40.	52%	35%	13%
Number System	15.	85%	15%			41.	56%	30%	14%
	16.	10%	89%	1%		42.	81%	16%	3%
	17.	82%	16%	2%		43.	35%	37%	28%
	18.	86%	11%	3%		44.	21%	48%	31%
	19.	78%	22%			45.	15%	38%	47%
	20.	80%	19%	1%		46.	34%	46%	20%
	21.	40%	51%	9%		47.	46%	40%	14%
	22.	57%	33%	10%		48.	43%	42%	15%
	23.	59%	38%	3%		49.	41%	38%	21%
	24.	12%	82%	6%		50.	39%	38%	23%

DISTRIBUTION OF SCORES FOR ARITHMETIC COMPREHENSIVE EXAM
MATC TECHNICAL MATHEMATICS STUDENTS (APRIL, 1969)

(Note: The table also includes the distribution of scores for the Arithmetic Pre-Test, in September, 1968, for these same 204 students.)

	Pre-Test Sept. 1968	Comp. Exam April, 1969
Mean	68.2%	89.8%
Median	72.0%	92.0%
N	204	204

NUMBER AND CUMULATIVE PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	Pre-Test Sept. 1968 N	Pre-Test Sept. 1968 Cum. %	Comp. Exam April 1969 N	Comp. Exam April 1969 Cum. %
50			10	4.9%
49	2	1.0%	20	14.7%
48	4	3.0%	29	28.9%
47	4	5.0%	29	43.1%
46	4	7.0%	23	54.4%
45	7	10.4%	22	65.2%
44	8	14.3%	19	74.5%
43	11	19.7%	11	79.9%
42	8	23.6%	7	83.3%
41	5	26.0%	8	87.2%
40	10	30.9%	5	89.6%
39	12	36.8%	3	91.1%
38	11	42.2%	8	95.0%
37	15	49.6%		
36	7	53.0%	1	95.5%
35	5	55.4%	3	97.0%
34	6	58.3%		
33	4	60.3%	2	98.0%
32	5	62.7%	2	99.0%
31	9	67.1%		
30	8	71.0%		
29	7	74.4%		
28	3	75.9%	1	99.5%
27	7	79.3%		
26	7	82.7%		
25	5	85.1%	1	100.0%
24	4	87.1%		
23	2	88.1%		
22	2	89.1%		
21	3	90.6%		
20	2	91.6%		
19	6	94.5%		
18	2	95.5%		
17	4	97.5%		
16	1	98.0%		
15				
14	2	99.0%		
9	1	99.5%		
5	1	100.0%		
	204		204	

ITEM ANALYSIS FOR ARITHMETIC COMPREHENSIVE EXAM MATC TECHNICAL MATHEMATICS STUDENTS (APRIL, 1969)

(Note: The table also includes the item analysis for the Arithmetic Pre-Test, in September, 1968, for these same 204 students.)

	Pre-Test Sept. 1968	Comp. Exam April, 1969
Mean	68.2%	89.8%
Median	72.0%	92.0%
N	204	204

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Pre-Test Sept. 1968	Comp. Exam April 1969	Gain	Topic	Item No.	Pre-Test Sept. 1968	Comp. Exam April 1969	Gain
Whole Numbers	1.	89%	97%	+ 8%	Number Sense	25.	83%	93%	+10%
	2.	92%	96%	+ 4%		26.	83%	91%	+ 8%
	3.	77%	90%	+13%		27.	65%	88%	+23%
	4.	80%	91%	+11%		28.	91%	99%	+ 8%
Decimals	5.	90%	97%	+ 7%	Fractions	29.	71%	94%	+23%
	6.	84%	93%	+ 9%		30.	86%	97%	+11%
	7.	78%	87%	+ 9%		31.	89%	100%	+11%
	8.	71%	89%	+18%		32.	65%	83%	+18%
						33.	93%	98%	+ 5%
Percents	9.	77%	89%	+12%		34.	80%	93%	+13%
	10.	79%	93%	+14%		35.	79%	98%	+19%
	11.	94%	96%	+ 2%		36.	61%	89%	+28%
	12.	79%	92%	+13%		37.	79%	98%	+19%
	13.	77%	91%	+14%		38.	57%	85%	+28%
	14.	59%	83%	+24%		39.	59%	95%	+36%
						40.	59%	99%	+40%
Number System	15.	87%	97%	+10%		41.	62%	96%	+34%
	16.	11%	34%	+23%		42.	83%	96%	+13%
	17.	86%	98%	+12%		43.	40%	89%	+49%
	18.	89%	99%	+10%		44.	23%	79%	+56%
	19.	82%	98%	+16%		45.	18%	78%	+60%
	20.	81%	95%	+14%		46.	42%	72%	+30%
	21.	41%	79%	+38%		47.	52%	92%	+40%
	22.	58%	91%	+33%		48.	49%	91%	+42%
	23.	67%	90%	+23%		49.	49%	89%	+40%
	24.	14%	65%	+51%		50	48%	80%	+32%

DISTRIBUTION OF SCORES FOR ARITHMETIC PRE-TEST
PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (SEPTEMBER, 1968)
(JUNIORS AND SENIORS)

Mean = 48.6%
Median = 46.0%
N = 138

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
50			
49			
48			
47			
46			
45			
44			
43			
42			
41	1	0.7%	
40	2	1.5%	2.2%
39	2	1.5%	3.7%
38	3	2.2%	5.9%
37	3	2.2%	8.1%
36	4	2.9%	11.0%
35	2	1.5%	12.5%
34	1	0.7%	13.2%
33	4	2.9%	16.1%
32	2	1.5%	17.6%
31	7	5.1%	22.7%
30	6	4.3%	27.0%
29	6	4.3%	31.3%
28	2	1.5%	32.8%
27	4	2.9%	35.7%
26	7	5.1%	40.8%
25	5	3.6%	44.4%
24	6	4.3%	48.7%
23	6	4.3%	53.0%
22	9	6.5%	59.5%
21	6	4.3%	63.8%
20	9	6.5%	70.3%
19	6	4.3%	74.6%
18	7	5.1%	79.7%
17	9	6.5%	86.2%
16	5	3.6%	89.8%
15	2	1.5%	91.3%
14	5	3.6%	94.9%
13	2	1.5%	96.4%
12	2	1.5%	97.9%
11	1	0.7%	98.6%
10	1	0.7%	99.3%
9	1	0.7%	100.0%
	N = 138	100.0%	

ITEM ANALYSIS FOR ARITHMETIC PRE-TEST
PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (SEPTEMBER, 1968)
(JUNIORS AND SENIORS)

Mean = 48.6%
Median = 46.0%
N = 138

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	%	Topic	Item No.	%
Whole Numbers	1.	94%	Number Sense	25.	83%
	2.	91%		26.	63%
	3.	64%		27.	30%
	4.	72%		28.	70%
Decimals	5.	75%	Fractions	29.	36%
	6.	64%		30.	72%
	7.	58%		31.	74%
	8.	44%		32.	51%
				33.	77%
Percents	9.	41%		34.	58%
	10.	33%		35.	64%
	11.	88%		36.	56%
	12.	35%		37.	74%
	13.	45%		38.	38%
	14.	28%		39.	30%
				40.	28%
Number System	15.	81%		41.	24%
	16.	10%		42.	77%
	17.	82%		43.	12%
	18.	86%		44.	5%
	19.	58%		45.	7%
	20.	73%		46.	17%
	21.	23%		47.	22%
	22.	44%		48.	21%
	23.	40%		49.	19%
	24.	5%		50.	8%

DISTRIBUTION OF SCORES FOR ARITHMETIC PRE-TEST PIUS XI HIGH SCHOOL ENTERING FRESHMEN (SEPTEMBER, 1968)

Overall Mean = 54.4%
Overall Median = 58.0%
N = 127

Score	Ability Levels				Overall	Percent of All Students	Cumulative Percent
	1	2	3 & 4	5			
50							
49							
48	1				1	0.8%	
47							
46							
45	1				1	0.8%	1.6%
44	5	1			6	4.7%	6.3%
43	3	1			4	3.1%	9.4%
42	1				1	0.8%	10.2%
41	3	1			4	3.1%	13.3%
40	3	2			5	3.9%	17.2%
39	5				5	3.9%	21.1%
38		1	1		2	1.6%	22.7%
37	1	2			3	2.4%	25.1%
36	2	2			4	3.1%	28.2%
35	2	2	2		6	4.7%	32.9%
34	2	5	1		8	6.3%	39.2%
33	1	2			3	2.4%	41.6%
32		3			3	2.4%	44.0%
31	2	3			5	3.9%	47.9%
30		1	1		2	1.6%	49.5%
29		1	2		3	2.4%	51.9%
28			1		1	0.8%	52.7%
27		1	2		3	2.4%	55.1%
26		2	5		7	5.4%	60.5%
25		1	3		4	3.1%	63.6%
24			1	1	2	1.6%	65.2%
23			1	1	2	1.6%	66.8%
22							
21			2		2	1.6%	68.4%
20			3	1	4	3.1%	71.5%
19				1	1	0.8%	72.3%
18				2	2	1.6%	73.9%
17			1	4	5	3.9%	77.8%
16							
15			1	2	3	2.4%	80.2%
14			1	1	2	1.6%	81.8%
13							
12			1	1	2	1.6%	83.4%
11			2	5	7	5.4%	88.8%
10				3	3	2.4%	91.2%
9			1	1	2	1.6%	92.8%
8							
7				3	3	2.4%	95.2%
6				2	2	1.6%	96.8%
5				2	2	1.6%	98.4%
4							
3							
2				2	2	1.6%	100.0%
N =	32	31	32	32	127	100.0%	

ITEM ANALYSIS (OVERALL) FOR ARITHMETIC PRE-TEST PIUS XI HIGH SCHOOL ENTERING FRESHMEN (SEPTEMBER, 1968)

Overall Mean = 54.4%
Overall Median = 58.0%
N = 127

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	%	Topic	Item No.	%
Whole Numbers	1.	84%	Number Sense	25.	68%
	2.	93%		26.	61%
	3.	76%		27.	42%
	4.	70%		28.	54%
Decimals	5.	83%	Fractions	29.	54%
	6.	67%		30.	72%
	7.	54%		31.	79%
	8.	53%		32.	62%
Percents	9.	51%		33.	72%
	10.	50%		34.	63%
	11.	71%		35.	83%
	12.	56%		36.	72%
	13.	56%		37.	82%
	14.	46%		38.	60%
Number System	15.	72%		39.	42%
	16.	9%		40.	47%
	17.	80%		41.	45%
	18.	84%		42.	78%
	19.	70%		43.	20%
	20.	66%		44.	9%
	21.	38%		45.	4%
	22.	40%		46.	23%
	23.	86%		47.	28%
	24.	0%		48.	30%
				49.	27%
				50.	18%

APPENDIX B-6

DISTRIBUTION OF SCORES FOR ARITHMETIC PRE-TEST
WEST DIVISION HIGH SCHOOL - TECHNICAL MATHEMATICS (SEPTEMBER, 1968)
(SOPHOMORES, JUNIORS, & SENIORS)

Mean = 33.1%
Median = 34.0%
N = 24

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
50			
49			
48			
47			
46			
45			
44			
43			
42			
41			
40			
39			
38			
37			
36			
35	1	4.2%	4.2%
34			
33			
32	1	4.2%	8.4%
31			
30			
29			
28			
27	2	8.3%	16.7%
26			
25			
24			
23			
22	1	4.2%	20.9%
21			
20	3	12.5%	33.4%
19	1	4.2%	37.6%
18	3	12.5%	50.1%
17			
16	1	4.2%	54.3%
15			
14			
13	2	8.3%	62.6%
12	1	4.2%	66.8%
11	2	8.3%	75.1%
10	2	8.3%	83.4%
9			
8			
7	2	8.3%	91.7%
6	1	4.2%	95.9%
5	1	4.2%	100.0%
	N = 24	100.0%	

APPENDIX B-6

ITEM ANALYSIS FOR ARITHMETIC PRE-TEST
WEST DIVISION HIGH SCHOOL - TECHNICAL MATHEMATICS (SEPTEMBER, 1968)
(SOPHOMORES, JUNIORS, & SENIORS)

Mean = 33.1%
Median = 34.0%
N = 24

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	%	Topic	Item No.	%
Whole Numbers	1.	81%	Number Sense	25.	50%
	2.	88%		26.	62%
	3.	65%		27.	35%
	4.	38%		28.	31%
Decimals	5.	69%	Fractions	29.	15%
	6.	38%		30.	58%
	7.	38%		31.	69%
	8.	23%		32.	38%
				33.	58%
Percents	9.	27%		34.	46%
	10.	27%		35.	65%
	11.	65%		36.	38%
	12.	23%		37.	54%
	13.	19%		38.	27%
	14.	19%		39.	15%
				40.	12%
Number System	15.	73%		41.	15%
	16.	15%		42.	54%
	17.	46%		43.	4%
	18.	46%		44.	4%
	19.	19%		45.	0%
	20.	50%		46.	0%
	21.	0%		47.	12%
	22.	12%		48.	0%
	23.	27%		49.	0%
	24.	8%		50.	8%

APPENDIX B-7

DISTRIBUTION OF SCORES FOR ARITHMETIC PRE-TEST ADMINISTERED TWO DIFFERENT TIMES
WEST DIVISION HIGH SCHOOL - TECHNICAL MATHEMATICS (1968-69)
(SOPHOMORES, JUNIORS, & SENIORS)

(Note: The same students, totaling 15, took both tests.)

	Sept., 1968	June, 1969
Mean	36.8%	65.2%
Median	38.0%	68.0%
N	15	15

NUMBER AND CUMULATIVE PERCENT OF STUDENTS ACHIEVING EACH SCORE

	Pre-Test Sept., 1968		Retest June, 1969	
Score	N	Cum. %	N	Cum. %
50				
49				
48				
47				
46				
45			1	6.7%
44			1	13.4%
43				
42				
41			1	20.1%
40			1	26.8%
39			2	40.1%
38				
37			1	46.7%
36				
35				
34			1	53.4%
33				
32	1	6.7%	1	60.0%
31				
30			1	66.7%
29				
28				
27	2	20.0%		
26			1	73.3%
25				
24			1	80.0%
23			1	86.6%
22	1	26.7%		
21			1	93.3%
20	3	46.7%		
19	1	53.4%		
18	2	66.7%		
17				
16	1	73.4%		
15				
14			1	100.0%
13				
12	1	80.0%		
11	1	86.7%		
10				
9				
8				
7	2	100.0%		
	15		15	

ITEM ANALYSIS FOR ARITHMETIC PRE-TEST
ADMINISTERED TWO DIFFERENT TIMES
WEST DIVISION HIGH SCHOOL - TECHNICAL MATHEMATICS (1968-69)
(SOPHOMORES, JUNIORS, & SENIORS)

(Note: The same students, totaling 15, took both tests.)

	Sept., 1968	June, 1969
Mean	36.8%	65.2%
Median	38.0%	68.0%
N	15	15

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Sept. 1968	June 1969
Whole Numbers	1.	100%	100%
	2.	87%	100%
	3.	60%	80%
	4.	47%	93%
Decimals	5.	73%	80%
	6.	47%	60%
	7.	33%	67%
	8.	13%	60%
Percents	9.	33%	47%
	10.	33%	40%
	11.	60%	80%
	12.	33%	60%
	13.	27%	40%
	14.	27%	20%
Number System	15.	73%	73%
	16.	13%	20%
	17.	53%	93%
	18.	53%	100%
	19.	13%	87%
	20.	53%	73%
	21.	0%	40%
	22.	13%	53%
	23.	33%	73%
	24.	13%	53%
Number Sense	25.	47%	60%
	26.	67%	87%
	27.	33%	73%
	28.	33%	80%
Fractions	29.	20%	40%
	30.	67%	73%
	31.	80%	93%
	32.	53%	73%
	33.	73%	93%
	34.	60%	87%
	35.	53%	73%
	36.	20%	53%
	37.	67%	73%
	38.	20%	47%
	39.	13%	80%
	40.	13%	87%
	41.	20%	87%
	42.	67%	80%
	43.	7%	67%
	44.	7%	40%
	45.	0%	27%
	46.	0%	47%
	47.	20%	47%
	48.	0%	27%
	49.	0%	40%
	50.	7%	33%

APPENDIX C

DATA FOR ALGEBRA PRE-TEST

C-1	Copy of Pre-Test: Algebra
C-2	Distribution of Scores for Algebra Pre-Test MATC Technical Mathematics Students (September, 1968)
	Item Analysis for Algebra Pre-Test MATC Technical Mathematics Students (September, 1968)
C-3	Distribution of Scores for Algebra Comprehensive Exam Administered Three Different Times MATC Technical Mathematics Students (1968-69)
	Item Analysis for Basic Algebra Comprehensive Exam Administered Three Different Times MATC Technical Mathematics Students (1968-69)
C-4	Distribution of Scores for Algebra Pre-Test Pius XI High School - Technical Mathematics (September, 1968) (Juniors and Seniors)
	Item Analysis for Algebra Pre-Test Pius XI High School - Technical Mathematics (September, 1968) (Juniors and Seniors)
C-5	Distribution of Scores for Algebra Pre-Test Administered Two Different Times (Sept., 1968 and Mar., 1969) Pius XI High School - Technical Mathematics (1968-69) (Juniors and Seniors)
	Item Analysis for Algebra Pre-Test Administered Two Different Times (Sept., 1968 and Mar., 1969) Pius XI High School - Technical Mathematics (1968-69) (Juniors and Seniors)
C-6	Distribution of Scores for Algebra Pre-Test West Division High School - Technical Mathematics (1968-69) (Sophomores, Juniors, & Seniors)
	Item Analysis for Algebra Pre-Test West Division High School - Technical Mathematics (1968-69) (Sophomores, Juniors, & Seniors)
C-7	Distribution of Scores for Algebra Pre-Test Administered Three Different Times (Sept. 1968, Apr. 1969, June 1969) West Division High School - Technical Mathematics (1968-69) (Sophomores, Juniors, & Seniors)
	Item Analysis for Algebra Pre-Test Administered Three Different Times (Sept. 1968, Apr. 1969, June 1969) West Division High School - Technical Mathematics (1968-69) (Sophomores, Juniors, & Seniors)
C-8	Distribution of Scores for Algebra Pre-Test For Technical Mathematics Class and Two Conventional Algebra Classes West Division High School (April, 1969)
	Item Analysis for Algebra Pre-Test For Technical Mathematics Class and Two Conventional Algebra Classes West Division High School (April, 1969)

PRE-TEST: ALGEBRA

Directions: Work each problem in the space provided. Show all necessary work. Write each answer in the answer box at the right.

Part I: Algebraic Operations

1. $(-5) - 9 = ?$

2. $3 + (-2) - 7 - (-9) = ?$

3. $(4)(-5)(0)(2) = ?$

4. $(-12) - (-3)(-2) = ?$

5. $5 - [(-3) + 7] = ?$

6. $7 - 2(1 - 5) = ?$

7. Complete: $\frac{10^{-2}}{10^{-3}} = 10^{?}$

8. Complete: $832{,}000 \times 10^{-4} = \underline{\ ?\ } \times 10^{1}$

9. Complete: $\frac{x}{3} + \frac{x}{2} = ?$

10. Combine: $\frac{r}{st} + \frac{2}{t} = ?$

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

Part II: Solution of Equations

Solve each of the following equations:

11. $2x + 8 = 0$

12. $13 = 4R - 7$

11. $x =$
12. $R =$

13. $8w - (2 + w) = 19$

14. $42 = 7 - 5(y + 1)$

13. $w =$

14. $y =$

15. $5 = R - 2(1 - 3R)$

16. $8 - (7 - h) = 5h$

15. $R =$

16. $h =$

17. $6 = \frac{7}{4x}$

18. $\frac{6w - 11}{w + 3} = 0$

17. $x =$

18. $w =$

19. $\frac{2}{3t} = \frac{t - 1}{t}$

20. $\frac{y}{2} - 5 = \frac{2y}{3}$

19. $t =$

20. $y =$

21. $\frac{7}{x} = \frac{9}{2x} - \frac{5}{6}$

22. $y - \frac{3y - 2}{2} = 5$

21. $x =$

22. $y =$

Part III: Formula Rearrangement

23. Solve for P: $t = \frac{W}{P}$

24. Solve for G: $M = K - G$

23. $P =$

24. $G =$

25. Solve for R: $A(G - R) = N$

26. Solve for M: $G = \frac{L - M}{P}$

25. $R =$

26. $M =$

27. Solve for V_1: $\frac{P_1}{P_2} = \frac{V_2}{V_1}$

28. Solve for P: $W + \frac{B}{P} = 0$

27. $V_1 =$

28. $P =$

29. Solve for A: $B = \frac{A}{1 - A}$

30. Solve for H: $\frac{1}{F} = \frac{1}{G} - \frac{1}{H}$

29. $A =$

30. $H =$

DISTRIBUTION OF SCORES FOR ALGEBRA PRE-TEST MATC TECHNICAL MATHEMATICS STUDENTS (SEPTEMBER, 1968)

Mean = 37.3%
Median = 30.0%
N = 471

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
30	1	0.2%	
29	4	0.8%	1.0%
28	3	0.6%	1.6%
27	6	1.3%	2.9%
26	6	1.3%	4.2%
25	5	1.1%	5.3%
24	14	3.0%	8.3%
23	16	3.4%	11.7%
22	9	1.9%	13.6%
21	16	3.4%	17.0%
20	11	2.3%	19.3%
19	10	2.1%	21.4%
18	12	2.5%	23.9%
17	10	2.1%	26.0%
16	12	2.6%	28.6%
15	13	2.8%	31.4%
14	14	3.0%	34.4%
13	20	4.2%	38.6%
12	19	4.0%	42.6%
11	13	2.8%	45.4%
10	18	3.8%	49.2%
9	21	4.5%	53.7%
8	18	3.8%	57.5%
7	30	6.4%	63.9%
6	35	7.4%	71.3%
5	37	7.9%	79.2%
4	26	5.5%	84.7%
3	26	5.5%	90.2%
2	23	4.9%	95.1%
1	10	2.1%	97.2%
0	13	2.8%	100.0%
	N = 471	100.0%	

ITEM ANALYSIS FOR ALGEBRA PRE-TEST
MATC TECHNICAL MATHEMATICS STUDENTS (SEPTEMBER, 1968)

Mean = 37.3%
Median = 30.0%
N = 471

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Correct	Incorrect	Not Attempted	Algebra Pre-Test Sept. 1967 Correct
Signed Numbers	1.	55%	44%	1%	52%
	2.	51%	46%	3%	54%
	3.	57%	40%	3%	52%
	4.	37%	60%	3%	---
	5.	71%	25%	4%	---
	6.	42%	55%	3%	---
Powers of Ten	7.	27%	59%	14%	---
	8.	12%	67%	21%	---
Algebraic Fractions	9.	44%	52%	4%	---
	10.	22%	70%	8%	---
Non-Fractional Equations	11.	82%	16%	2%	---
	12.	84%	13%	3%	---
	13.	61%	27%	12%	68%
	14.	40%	47%	13%	44%
	15.	50%	30%	20%	57%
	16.	46%	26%	28%	---
Fractional Equations	17.	32%	33%	35%	40%
	18.	15%	42%	43%	18%
	19.	18%	34%	48%	21%
	20.	22%	31%	47%	25%
	21.	18%	29%	53%	20%
	22.	10%	42%	48%	---
Formula Rearrangement	23.	46%	34%	20%	53%
	24.	53%	30%	17%	56%
	25.	27%	41%	32%	---
	26.	29%	35%	36%	34%
	27.	31%	26%	43%	38%
	28.	23%	37%	40%	---
	29.	5%	41%	54%	8%
	30.	7%	44%	49%	9%

DISTRIBUTION OF SCORES FOR BASIC ALGEBRA COMPREHENSIVE EXAM ADMINISTERED THREE DIFFERENT TIMES MATC TECHNICAL MATHEMATICS STUDENTS (1968-69)

(Note: The same students, totaling 196,
took all three tests.)

	Pre-Test Sept., 1968	Retest Feb., 1969	Comp. Exam April, 1969
Mean	43.6%	82.8%	90.6%
Median	40.0%	86.7%	93.3%
N	196	196	196

NUMBER AND CUMULATIVE PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	Pre-Test Sept. 1968 N	Pre-Test Sept. 1968 Cum. %	Retest Feb. 1969 N	Retest Feb. 1969 Cum. %	Comp. Exam April 1969 N	Comp. Exam April 1969 Cum. %
30	1	0.5%	13	6.6%	33	16.8%
29	2	1.5%	27	20.4%	40	37.2%
28	2	2.5%	27	34.2%	41	58.1%
27	4	4.5%	22	45.4%	27	71.9%
26	5	7.1%	18	54.6%	12	78.0%
25	1	7.6%	15	62.3%	12	84.1%
24	7	11.2%	14	69.4%	9	88.7%
23	9	15.8%	13	76.0%	8	92.6%
22	4	17.8%	7	79.6%	5	95.4%
21	10	22.9%	7	83.3%	5	98.0%
20	5	25.5%	5	85.9%	1	98.5%
19	6	28.6%	3	87.4%	2	99.5%
18	8	32.6%	12	93.5%	1	100.0%
17	4	34.6%	3	95.0%		
16	6	37.7%	3	96.5%		
15	4	39.7%	2	97.5%		
14	6	42.8%	2	98.5%		
13	10	47.9%	1	99.0%		
12	10	53.0%	1	99.5%		
11	5	55.6%				
10	8	59.7%				
9	9	64.3%	1	100.0%		
8	5	66.9%				
7	14	74.0%				
6	15	81.7%				
5	14	88.8%				
4	4	90.8%				
3	6	93.9%				
2	7	97.5%				
1	3	99.0%				
0	2	100.0%				
	196		196		196	

APPENDIX C-3

ITEM ANALYSIS FOR BASIC ALGEBRA COMPREHENSIVE EXAM
ADMINISTERED THREE DIFFERENT TIMES
MATC TECHNICAL MATHEMATICS STUDENTS (1968-69)

(Note: The same students, totaling 196,
took all three tests.)

	Pre-Test Sept., 1968	Retest Feb., 1969	Comp. Exam April, 1969
Mean	43.6%	82.8%	90.6%
Median	40.0%	86.7%	93.3%
N	196	196	196

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Pre-Test Sept. 1968	Retest Feb. 1969	Comp. Exam April 1969	Overall Gain From Pre-Test to Comp. Exam
Signed Numbers	1.	65%	99%	98%	+33%
	2.	60%	95%	98%	+38%
	3.	62%	87%	98%	+36%
	4.	41%	86%	85%	+44%
	5.	80%	95%	96%	+16%
	6.	50%	86%	91%	+41%
Powers of Ten	7.	32%	90%	89%	+57%
	8.	19%	82%	85%	+66%
Algebraic Fractions	9.	52%	76%	91%	+39%
	10.	27%	73%	81%	+54%
Non-Fractional Equations	11.	86%	99%	98%	+12%
	12.	59%	98%	99%	+40%
	13.	68%	94%	95%	+27%
	14.	46%	84%	94%	+48%
	15.	59%	88%	92%	+33%
	16.	56%	90%	97%	+41%
Fractional Equations	17.	43%	87%	99%	+56%
	18.	20%	52%	69%	+49%
	19.	23%	70%	86%	+63%
	20.	27%	82%	90%	+63%
	21.	24%	77%	88%	+64%
	22.	10%	44%	60%	+50%
Formula Rearrangement	23.	55%	95%	99%	+44%
	24.	64%	91%	98%	+34%
	25.	33%	74%	87%	+54%
	26.	33%	81%	95%	+62%
	27.	38%	93%	97%	+59%
	28.	31%	74%	89%	+58%
	29.	7%	69%	93%	+86%
	30.	9%	70%	79%	+70%

DISTRIBUTION OF SCORES FOR ALGEBRA PRE-TEST
PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (SEPTEMBER, 1968)
(JUNIORS AND SENIORS)

Mean = 12.4%
Median = 13.3%
N = 139

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
30			
29			
28			
27			
26			
25			
24			
23			
22			
21			
20			
19			
18			
17			
16			
15			
14			
13			
12	1	0.7%	0.7%
11			
10			
9			
8	7	5.0%	5.7%
7	9	6.5%	12.2%
6	12	8.6%	20.8%
5	18	13.0%	33.8%
4	26	18.7%	52.5%
3	18	13.0%	65.5%
2	27	19.4%	84.9%
1	14	10.1%	95.0%
0	7	5.0%	100.0%

ITEM ANALYSIS FOR ALGEBRA PRE-TEST
PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (SEPTEMBER, 1968)
(JUNIORS AND SENIORS)

Mean = 12.4%
Median = 13.3%
N = 139

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	%
Signed Numbers	1	34%
	2	19%
	3	22%
	4	12%
	5	47%
	6	9%
Powers of Ten	7	7%
	8	1%
Algebraic Fractions	9	20%
	10	0%
Non-Fractional Equations	11	63%
	12	66%
	13	37%
	14	4%
	15	16%
	16	0%
Fractional Equations	17	1%
	18	1%
	19	0%
	20	1%
	21	0%
	22	0%
Formula Rearrangement	23	6%
	24	8%
	25	0%
	26	0%
	27	0%
	28	1%
	29	0%
	30	0%

DISTRIBUTION OF SCORES FOR ALGEBRA PRE-TEST ADMINISTERED TWO DIFFERENT TIMES (SEPT., 1968 AND MAR., 1969)
PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (1968-69)
(JUNIORS AND SENIORS)

(Note: The same students, totaling 127, took each test.)

	Pre-Test Sept., 1968	Post-Test Mar., 1969
Mean	12.6%	68.1%
Median	13.3%	70.0%
N	127	127

NUMBER AND CUMULATIVE PERCENT OF STUDENTS ACHIEVING EACH SCORE

	Pre-Test Sept. 1968		Post-Test Mar. 1969	
Score	N	Cum. %	N	Cum. %
30			1	0.8%
29			5	4.7%
28			7	10.2%
27			9	17.3%
26			6	22.0%
25			8	28.3%
24			6	33.0%
23			14	44.0%
22			3	46.4%
21			11	55.1%
20			7	60.6%
19			6	65.3%
18			7	70.8%
17			3	73.2%
16			9	80.3%
15			4	83.4%
14			3	85.8%
13			4	88.9%
12	1	0.8%	5	92.8%
11			2	94.4%
10				
9			2	96.0%
8	6	5.5%	2	97.6%
7	9	12.6%	1	98.4%
6	10	20.5%	1	99.2%
5	18	34.7%	1	100.0%
4	25	54.4%		
3	17	67.8%		
2	23	85.9%		
1	12	95.3%		
0	6	100.0%		

ITEM ANALYSIS FOR ALGEBRA PRE-TEST ADMINISTERED TWO DIFFERENT TIMES (SEPT., 1968 AND MAR., 1969) PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (1968-69)

(JUNIORS AND SENIORS)

(Note: The same students, totaling 127, took each test.)

	Pre-Test Sept., 1968	Post-Test Mar., 1969
Mean	12.6%	68.1%
Median	13.3%	70.0%
N	127	127

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Pre-Test Sept., 1968	Post-Test Mar., 1969	Gain
Signed Numbers	1	35%	99%	+64%
	2	18%	95%	+77%
	3	24%	83%	+59%
	4	13%	78%	+65%
	5	50%	88%	+38%
	6	8%	65%	+57%
Powers of Ten	7	7%	67%	+60%
	8	0%	58%	+58%
Algebraic Fractions	9	20%	71%	+51%
	10	0%	66%	+66%
Non-Fractional Equations	11	66%	94%	+28%
	12	69%	96%	+27%
	13	35%	81%	+46%
	14	5%	58%	+53%
	15	15%	66%	+51%
	16	0%	61%	+61%
Fractional Equations	17	1%	74%	+73%
	18	1%	39%	+38%
	19	0%	47%	+47%
	20	1%	57%	+56%
	21	0%	48%	+48%
	22	0%	19%	+19%
Formula Rearrangement	23	5%	86%	+81%
	24	9%	72%	+63%
	25	0%	61%	+61%
	26	0%	72%	+72%
	27	0%	84%	+84%
	28	1%	47%	+46%
	29	0%	50%	+50%
	30	0%	62%	+62%

DISTRIBUTION OF SCORES FOR ALGEBRA PRE-TEST
WEST DIVISION HIGH SCHOOL - TECHNICAL MATHEMATICS (1968-69)
(SOPHOMORES, JUNIORS, & SENIORS)

Mean = 19.7%
Median = 16.7%
N = 24

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
30			
29			
28			
27			
26			
25			
24			
23			
22			
21			
20			
19			
18			
17			
16			
15			
14	1	4.2%	4.2%
13			
12			
11			
10	4	16.6%	20.8%
9	1	4.2%	25.0%
8	2	8.3%	33.3%
7	2	8.3%	41.6%
6	1	4.2%	45.8%
5	3	12.5%	58.3%
4	1	4.2%	62.5%
3	6	25.0%	87.5%
2	3	12.5%	100.0%

ITEM ANALYSIS FOR ALGEBRA PRE-TEST
WEST DIVISION HIGH SCHOOL - TECHNICAL MATHEMATICS (1968-69)
(SOPHOMORES, JUNIORS, & SENIORS)

Mean = 19.7%
Median = 16.7%
N = 24

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Pre-Test Sept., 1968
Signed Numbers	1	42%
	2	42%
	3	75%
	4	12%
	5	58%
	6	17%
Powers of Ten	7	29%
	8	0%
Algebraic Fractions	9	25%
	10	0%
Non-Fractional Equations	11	75%
	12	88%
	13	42%
	14	12%
	15	46%
	16	21%
Fractional Equations	17	4%
	18	4%
	19	0%
	20	0%
	21	0%
	22	0%
Formula Rearrangement	23	4%
	24	12%
	25	0%
	26	0%
	27	0%
	28	0%
	29	0%
	30	0%

APPENDIX C-7

DISTRIBUTION OF SCORES FOR ALGEBRA PRE-TEST ADMINISTERED THREE DIFFERENT TIMES (SEPT. 1968, APR. 1969, JUNE 1969) WEST DIVISION HIGH SCHOOL - TECHNICAL MATHEMATICS (1968-69) (SOPHOMORES, JUNIORS, & SENIORS)

(Note: The same students, totaling 15, took all three tests.)

	Pre-Test Sept. 1968	Retest Apr. 1969	Retest June 1969
Mean	23.3%	63.8%	82.7%
Median	26.7%	73.3%	90.0%
N	15	15	15

NUMBER AND CUMULATIVE PERCENT OF STUDENTS ACHIEVING EACH SCORE

	Pre-Test Sept. 1968		Retest Apr. 1969		Retest June, 1969	
Score	N	Cum. %	N	Cum. %	N	Cum. %
30					2	13.3%
29					2	26.6%
28					3	46.6%
27			1	6.7%	2	59.9%
26						
25			1	13.4%		
24			3	33.4%	1	66.6%
23			1	40.1%		
22			2	53.3%	1	73.3%
21						
20			1	60.0%	1	80.0%
19						
18					1	86.7%
17			2	73.2%		
16					2	100.0%
15			1	79.9%		
14	1	6.7%	1	86.6%		
13						
12						
11						
10	3	26.7%				
9	1	33.4%				
8	3	53.4%				
7			1	93.3%		
6	1	60.0%	1	100.0%		
5	2	73.3%				
4	1	80.0%				
3	2	93.3%				
2	1	100.0%				

ITEM ANALYSIS FOR ALGEBRA PRE-TEST
ADMINISTERED THREE DIFFERENT TIMES (SEPT. 1968, APR. 1969, JUNE 1969)
WEST DIVISION HIGH SCHOOL - TECHNICAL MATHEMATICS (1968-69)
(SOPHOMORES, JUNIORS, & SENIORS)

(Note: The same students, totaling 15, took all three tests.)

	Pre-Test Sept. 1968	Retest Apr. 1969	Retest June 1969
Mean	23.3%	63.8%	82.7%
Median	26.7%	73.3%	90.0%
N	15	15	15

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Pre-Test Sept. 1968	Retest Apr. 1969	Retest June 1969	Gain From Sept. 1968 to June 1969
Signed Numbers	1.	53%	87%	100%	+ 47%
	2.	53%	87%	93%	+ 40%
	3.	73%	100%	100%	+ 27%
	4.	13%	60%	87%	+ 74%
	5.	60%	80%	93%	+ 33%
	6.	27%	67%	93%	+ 66%
Powers of Ten	7.	40%	47%	73%	+ 33%
	8.	0%	33%	73%	+ 73%
Algebraic Fractions	9.	33%	47%	93%	+ 60%
	10.	0%	67%	100%	+100%
Non-Fractional Equations	11.	80%	93%	100%	+ 20%
	12.	100%	93%	100%	0%
	13.	47%	73%	80%	+ 33%
	14.	20%	53%	73%	+ 53%
	15.	47%	73%	73%	+ 26%
	16.	27%	47%	80%	+ 53%
Fractional Equations	17.	7%	60%	100%	+ 93%
	18.	7%	27%	80%	+ 73%
	19.	0%	60%	80%	+ 80%
	20.	0%	53%	80%	+ 80%
	21.	0%	53%	87%	+ 87%
	22.	0%	13%	47%	+ 47%
Formula Rearrangement	23.	0%	87%	93%	+ 93%
	24.	13%	80%	73%	+ 60%
	25.	0%	67%	73%	+ 73%
	26.	0%	73%	80%	+ 80%
	27.	0%	80%	87%	+ 87%
	28.	0%	60%	73%	+ 73%
	29.	0%	33%	60%	+ 60%
	30.	0%	60%	53%	+ 53%

APPENDIX C-8

DISTRIBUTION OF SCORES FOR ALGEBRA PRE-TEST FOR TECHNICAL MATHEMATICS CLASS AND TWO CONVENTIONAL ALGEBRA CLASSES WEST DIVISION HIGH SCHOOL (APRIL, 1969)

	Tech Math Class	Algebra I Class (First Semester)	Algebra I Class (Second Semester)
Mean	63.8%	21.8%	38.0%
Median	73.3%	20.0%	35.0%
N	15	19	24

NUMBER AND CUMULATIVE PERCENT OF STUDENTS ACHIEVING EACH SCORE

	Tech Math Class April 1969		Algebra I Class (First Semester) April 1969		Algebra I Class (Second Semester) April 1969	
Score	N	Cum. %	N	Cum. %	N	Cum. %
30						
29						
28						
27	1	6.7%				
26						
25	1	13.4%			1	4.2%
24	3	33.4%				
23	1	40.1%			1	8.4%
22	2	53.3%			1	12.6%
21						
20	1	60.0%				
19					3	25.0%
18						
17	2	73.2%			1	29.2%
16					1	33.4%
15	1	79.9%	1	5.3%	1	37.6%
14	1	86.6%			1	41.8%
13						
12			1	10.6%	2	50.1%
11			1	15.9%		
10						
9			3	31.6%	1	54.3%
8			1	36.9%		
7	1	93.3%	2	47.4%	2	62.6%
6	1	100.0%	4	68.4%	3	75.0%
5					3	87.4%
4			2	78.9%	1	91.6%
3			1	84.2%		
2						
1			2	94.7%	1	95.8%
0			1	100.0%	1	100.0%

ITEM ANALYSIS FOR ALGEBRA PRE-TEST FOR TECHNICAL MATHEMATICS CLASS AND TWO CONVENTIONAL ALGEBRA CLASSES WEST DIVISION HIGH SCHOOL (APRIL, 1969)

	Tech Math Class	Algebra I Class (First Semester)	Algebra I Class (Second Semester)
Mean	63.8%	21.8%	38.0%
Median	73.3%	20.0%	35.0%
N	15	19	24

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Tech Math Class April 1969	Algebra I Class (First Semester) April 1969	Algebra I Class (Second Semester) April 1969
Signed Numbers	1.	81%	63%	71%
	2.	88%	47%	58%
	3.	100%	74%	79%
	4.	56%	32%	29%
	5.	81%	74%	62%
	6.	62%	37%	54%
Powers of Ten	7.	44%	5%	42%
	8.	38%	0%	12%
Algebraic Fractions	9.	50%	16%	29%
	10.	62%	10%	25%
Non-Fractional Equations	11.	88%	74%	75%
	12.	94%	79%	92%
	13.	75%	47%	50%
	14.	56%	10%	42%
	15.	75%	53%	62%
	16.	50%	16%	50%
Fractional Equations	17.	62%	0%	46%
	18.	31%	5%	29%
	19.	56%	5%	25%
	20.	56%	0%	29%
	21.	56%	0%	42%
	22.	12%	0%	12%
Formula Rearrangement	23.	81%	5%	42%
	24.	81%	0%	29%
	25.	62%	0%	21%
	26.	69%	0%	17%
	27.	75%	0%	12%
	28.	56%	0%	17%
	29.	31%	0%	4%
	30.	56%	0%	0%

APPENDIX D

DESCRIPTION OF COURSE CONTENT TECHNICAL MATHEMATICS I AND II (1968-69) MILWAUKEE AREA TECHNICAL COLLEGE

APPENDIX D

DESCRIPTION OF COURSE CONTENT TECHNICAL MATHEMATICS I AND II (1968-69)

There are five major sections of the course. They are:

I. ALGEBRA
II. CALCULATIONS AND SLIDE RULE OPERATIONS
III. GRAPHING
IV. GEOMETRY AND TRIGONOMETRY
V. LOGARITHMS AND EXPONENTIALS

The specific content of each major section is described in this appendix. Each section subheading consists of a programmed booklet.

SECTION I: ALGEBRA

The content of this section includes: operations with fractions, the solution of non-fractional and fractional equations and formulas, systems of equations and formulas, radicals and radical equations and formulas, quadratic equations and formulas.

(1) ALGEBRA I: SIGNED NUMBERS

Signed numbers on the number line.
Addition of signed numbers.
Subtraction of signed numbers.
Multiplication of signed numbers.
Commutative principle of addition and multiplication.
Combined operations involving addition, subtraction, and multiplication of signed numbers.

(2) ALGEBRA II: NON-FRACTIONAL EQUATIONS I

Note: All equations are non-fractional, first-degree, single-variable equations whose roots are integers.

Meaning of equation and root.
Intuitive solution of basic equations.
Distributive principle and its use in combining terms containing the same letter.
Interchange principle for equations.
Oppositing principle for equations.
Addition axiom for equations.
Formal strategies for solving equations.

(3) ALGEBRA III: NON-FRACTIONAL EQUATIONS II

Note: All equations are non-fractional, first-degree, single-variable equations whose roots are integers.

Identity principle of multiplication.
Opposite principle of multiplication.
Solving equations using the identity and opposite principles of multiplication.
Checking the root of an equation.
Solving equations containing groupings and instances of the distributive principle.
Formal strategies for solving equations.

(4) ALGEBRA IV: MULTIPLICATION AND DIVISION OF FRACTIONS

Meaning of fractions.
Multiplying two fractions.
Multiplying a fraction and a non-fraction.
Factoring fractions.
The principle of dividing a quantity by itself.
Writing a fraction in higher terms.
Reducing a fraction to lowest terms.
Multiplication of signed fractions.
Concept of reciprocals.
Dividing two quantities by converting to multiplication.
Dividing signed numbers and fractions.

(5) ALGEBRA V: ADDITION, SUBTRACTION, AND COMBINED OPERATIONS WITH FRACTIONS

Adding fractions having like denominators.
Adding fractions having unlike denominators.
Adding a fraction and a non-fraction.
Converting a mixed number to a fraction, and vice versa.
Adding mixed numbers.
Adding signed fractions and signed mixed numbers.
Subtracting fractions and mixed numbers.
Multiplying and dividing mixed numbers.
Simplifying complicated expressions involving addition, subtraction, multiplication, and division of fractions.
Checking non-fractional equations with fractional roots.
Checking fractional equations with integral or fractional roots.

(6) ALGEBRA VI: FRACTIONAL ROOTS AND FRACTIONAL EQUATIONS

Note: All equations are non-fractional and fractional, first-degree, single-variable equations. The non-fractional equations have roots which are fractions. The fractional equations have roots which are either integers or fractions.

Multiplication axiom for equations.
Review of principles and axioms for solving equations.
Clearing fractions in fractional equations by means of the multiplication axiom and the distributive principle.
Solving fractional equations containing a single fraction.
Solving more-complicated fractional equations.

(7) ALGEBRA VII: INTRODUCTION TO GRAPHING

(A description of the content of this booklet is given in Section III: GRAPHING.)

(8) ALGEBRA VIII: LITERAL FRACTIONS

Multiplication of literal fractions.
Factoring literal fractions.
Writing literal fractions in equivalent forms.
Reducing simple literal fractions to lowest terms.
Division of literal fractions.
Addition of literal fractions.
Subtraction of literal fractions.
Reversing the process for adding or subtracting literal fractions.
Reducing complicated literal fractions to lowest terms.
Simplifying complicated literal fractions by performing indicated operations.

(9) ALGEBRA IX: FORMULA REARRANGEMENT

Note: In all formulas rearranged in this unit, the variable solved for is of first-degree.

Review of basic principles of solving equations.
Definition of literal equations and formulas.
Rearranging formulas having one term on each side:
  Both terms non-fractional.
  One term fractional, and one term non-fractional.
  Both terms fractional.
Rearranging formulas having more than one term on one side.
Rearranging formulas requiring use of the distributive principle.
Writing solutions in preferred forms.

(10) SYSTEMS OF EQUATIONS

Meaning of a system of equations.
Meaning of the solution of a system of equations.
Graphical solution of a system of two equations.
Algebraic solution of a system of two equations.
Solving systems of two equations which contain decimals or fractions or instances of the distributive principle.
Applied problems involving systems of equations.
Algebraic solution of a system of three equations.
Meaning of a system of formulas.
Deriving a new formula from a system of two formulas by eliminating a common variable.
Deriving a new formula from a system of three formulas by eliminating a common variable.

(11) RADICALS AND RADICAL EQUATIONS

Note: All radicals dealt with in this booklet are square root radicals.

Operations with radicals: multiplication, factoring, squaring, addition, subtraction, division, simplification, and rationalizing denominators.
Meaning of radical equations.
The squaring axiom for equations.
Solving radical equations.
Rearranging formulas involving radicals, including solving for a variable under the radical.
The square root axiom for equations.
Solving for a squared variable in a formula.
Deriving a new formula from a system of formulas containing radicals or squared letters.

(12) QUADRATIC EQUATIONS

Multiplying two binomials
Factoring a binomial.
Meaning of quadratic equation.
Solving quadratic equations by the factoring method.
Standard form of quadratic equations.
Solving quadratic equations by the quadratic formula.

SECTION II: CALCULATIONS

The content of this section consists of number system structure, powers of ten, estimation of numerical answers, and slide rule calculations. The slide rule work involves multiplication, division, combined multiplication and division, squaring and square root, and cubing and cube root.

(1) CALCULATIONS I: NUMBER SYSTEM AND NUMBER SENSE

Layout of decimal number system.
Concept of place-value of a digit.
Place-names of digits.
Naming whole and non-whole numbers.
Comparing the sizes of whole numbers.
Position of non-whole numbers on scales.
Converting decimal fractions to regular decimal numbers, and vice versa.
Comparing the sizes of non-whole numbers.

(2) CALCULATIONS II: POWERS OF TEN

Note: All powers of ten used in this unit have integral exponents.

Meaning of powers of ten.
Multiplication of powers of ten.
Division of powers of ten.
Meaning of 10^0.
Reciprocals of powers of ten.
Combined multiplication and division of powers of ten.
Laws of exponents.
Multiplication and division of regular numbers by powers of ten.
Meaning of standard notation.
Expressing regular numbers in standard notation.
Relationship between decimal number system and powers of ten.
Writing a regular number in power-of-ten form with a specified power of ten.
Comparing the sizes of numbers written in power-of-ten form.

(3) CALCULATIONS III: ROUNDING AND ROUGH ESTIMATION

Rounding a whole number to a specified place, and to a specified number of digits.
Rounding a non-whole number to a specified place, and to a specified number of digits.
Estimating products by rounding.
Checking the sensibleness of a given product.
Estimating quotients by rounding.
Checking the sensibleness of a given quotient.
Using estimation to place the decimal point in the digits of a slide rule product.
Using estimation to place the decimal point in the digits of a slide rule quotient.

(4) CALCULATIONS IV: INTRODUCTION TO SLIDE RULE

Slide rule parts: frame, slide, hairline.
Reading and setting numbers on the C and D scales.
Multiplication procedure for the slide rule.
Multiplying two numbers, one of which lies between 1 and 10.
Multiplying three or four numbers each of which lies between 1 and 10.
Division procedure for the slide rule.
Dividing two numbers, with the divisor a number between 1 and 10.
Applied problems in multiplication and division.

(5) CALCULATIONS V: SLIDE RULE MULTIPLICATION AND DIVISION

Estimating a product of two numbers by means of powers of ten.
Using an estimated product of two numbers, obtained by means of powers of ten, to place the decimal point in the digits of a slide rule product.
Estimating a product of two numbers by means of the "decimal point shift" method.
Multiplying two numbers of any size on the slide rule.
Estimating a product of three or more numbers by means of powers of ten.
Multiplying three or more numbers of any size on the slide rule.
Estimating the quotient of two numbers, with the divisor a number between 1 and 10.
Estimating a quotient of two numbers by means of the "decimal point shift" method.
Estimating a quotient of two numbers by means of powers of ten.
Dividing two numbers of any size on the slide rule.
Estimating answers to combined multiplication and division problems by means of powers of ten.
Using the slide rule to work problems in combined multiplication and division.

(6) CALCULATIONS VI: SLIDE RULE POWERS AND ROOTS

Reading and setting numbers on the A and B scales.
Squaring numbers on the slide rule.
Estimating answers to squaring problems by means of powers of ten.
Finding square roots on the slide rule.
Estimating answers to square root problems by means of powers of ten.
Using the grouping method to shorten the square root process.
Cubing numbers on the slide rule.
Reading and setting numbers on the K scale.
Estimating answers to cubing problems by means of powers of ten.
Using the grouping method to shorten the cube root process.
Finding approximate square roots mentally.
Performing calculations with signed numbers (decimals and whole numbers), needed later in logarithmic work.

(7) TECHNICAL MEASUREMENT

Definition of percent.
Converting percents to fractions and decimals.
Converting fractions and decimals to percents.
Basic percent formula and related problems.

Approximate nature of measurement.
Concept of precision.
Upper and lower values of reported measurements.
Reading measurement scales.
Concept of absolute error.
Rounding numbers.
Addition and subtraction of measurements.
Concept of relative error and accuracy.
Concept of significant digits.
Expressing accuracy in terms of significant digits.
Multiplication and division of measurements.
Concept of error in measurement.

SECTION III: GRAPHING

The content of this section consists of the rectangular coordinate system, graphing linear and non-linear equations and formulas, reading scientific and technical graphs, the concept of slope and changes in variables, and determining the equation or formula of a graphed line.

(1) GRAPHING I: INTRODUCTION TO GRAPHING

Note: This booklet is entitled "Algebra VII."

Meaning of solutions of two-variable equations.
Preparing tables of solutions for two-variable equations.
Layout of rectangular coordinate system.
Plotting and reading points on rectangular coordinate system.
Meaning of abscissa, ordinate, ordered pair, origin, quadrant.
Graphing two-variable linear equations.
Graphing two-variable non-linear equations.
Graphing two-variable formulas.
Graphing three-variable formulas, holding one variable constant.
Reading scientific and technical graphs.

(2) GRAPHING II: STRAIGHT LINE AND SLOPE

Note: In the following, the equations contain two variables called x and y.

General straight line equation: $y = mx + b$.
Determining whether a given equation is a straight line.
Determining the intercepts of a given equation.
Obtaining the vertical intercept from $y = mx + b$.
Representing horizontal and vertical changes by vectors.
Definition of slope of a line: $m = \frac{\Delta y}{\Delta x}$
Obtaining the slope of a line from $y = mx + b$.
Determining the slope of a line through two given points.
Determining the equation of a line through two given points.
Determining the equation of a graphed line.
Equations, graphs, and slopes of lines through the origin.
Equations, graphs, and slopes of horizontal, vertical, and parallel lines.
Using the slope formula to determine changes in variables.

Note: In the following, the equations are formulas which contain two variables. The letters x and y are not used for the variables.

Determining the intercepts of a given formula.
Definition of slope of a line: $\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}}$
Slope-intercept form of a formula.
Determining whether a given formula is a straight line.
Linear graphs of formulas which pass through the origin.
Using the slope formula to determine changes in variables.
Determining the slope of a curvilinear graph at various points on the graph.

SECTION IV: GEOMETRY AND TRIGONOMETRY

The content of this section consists of properties of basic geometric figures, definitions of the trigonometric ratios or functions, use of trigonometric tables, solution of right and oblique triangles, vectors and vector operations, complex numbers, sine-wave analysis, applied problems in geometry and trigonometry, basic trigonometric identities, radian measure of angles, and inverse trigonometric notation.

(1) TRIANGLES AND TRIGONOMETRY

Areas and properties of rectangles, squares, and parallelograms.
Volumes and surface areas of cubes and boxes.
Areas of general triangles.
Sum of the three angles of any triangle equals 180°.
Relationship between the three sides of a right triangle, expressed in the Pythagorean Theorem.
Using scale drawings to find an unknown side of a right triangle.
Properties of similar right triangles.
Definition of tangent of an angle.
Definition of sine of an angle.
Definition of cosine of an angle.
Table of numerical values of sine, cosine, and tangent.
Finding unknown sides and angles of right triangles by means of sine, cosine, and tangent.
Solving applied problems involving right triangles by means of sine, cosine, and tangent.

(2) VECTORS

Horizontal, vertical, and slant vectors on the coordinate system.
Components of a vector.
Reference angle of a vector.
Designating a vector by its length and reference angle.
Definition of sine, cosine, and tangent of reference angles in any quadrant.
Calculating the length and reference angle of a vector, given its components.
Calculating the components of a vector, given its length and reference angle.
Finding the sum (resultant) of two vectors.
Finding the sum (resultant) of more than two vectors.
Condition of equilibrium in a system of vectors.
Finding the equilibrant of a system of vectors.

(3) GENERAL ANGLES

Standard-position angles on the coordinate system.
Relationship between reference angles and standard-position angles.
Definitions of sine, cosine, and tangent for angles between 90° and 360°.
Definitions of sine, cosine, and tangent for 0°, 90°, 180°, 270°, and 360°.
Graph of $y = \sin \theta$ between 0° and 360°.
Definition of sine, cosine, and tangent for angles greater than 360°.
Definitions of sine, cosine, and tangent for negative angles.
Use of slide rule in finding numerical values of sine, cosine, and tangent.

(4) OBLIQUE TRIANGLES

Definition of oblique triangle.
Solving an oblique triangle by resolving it into right triangles.
Defining and proving the law of sines.
Solving acute oblique triangles by the law of sines.
Defining and proving the law of cosines.
Solving acute oblique triangles by the law of cosines.
Discriminating whether to use the law of sines or the law of cosines when solving an acute oblique triangle.
Finding the sine and cosine of an obtuse angle.
Solving obtuse oblique triangles by the law of sines.
Solving obtuse oblique triangles by the law of cosines.
Discriminating whether to use the law of sines or the law of cosines when solving an obtuse oblique triangle.
Finding the length of the resultant of two vectors by the law of cosines.
Finding angles between vectors by the law of sines.

(5) COMPLEX NUMBERS

Concept of real and imaginary numbers.
Square root of a negative number.
General complex number, $a + bj$, where $j = \sqrt{-1}$.
Representing vectors by complex numbers, and vice versa.
Finding vector resultants by means of complex numbers.
Addition and subtraction of complex numbers.
Polar coordinate representation of vectors.
Converting complex numbers to polar coordinate form, and vice versa.
Multiplying vectors in complex number form, and in polar coordinate form.
Dividing vectors in complex number form, and in polar coordinate form.
Simplifying complicated vector problems by operations with complex numbers.

(6) SINE WAVE ANALYSIS

Review of the graph of $y = \sin \theta$.
Finding solution pairs for various sine wave equations.
Fundamental sine wave equation: $y = A \sin \theta$
Determining sine wave amplitude.
General sine wave equation: $y = A \sin(\theta \pm \nu)$
Shifting a sine wave graph to the right or left.
Concept of in-phase and out-of-phase.
Sine wave harmonics equation: $y = A \sin k\theta$
Sketching graphs of fundamental sine waves and sine wave harmonics.
Sine waves having negative amplitudes.
Graph of cosine equation: $y = A \cos \theta$
Degree and radian measurement of angles.
Converting radians to degrees, and vice versa.
Graph of $y = \sin \theta$ when θ is measured in radians.

(7) GEOMETRY AND APPLIED TRIGONOMETRY

Definition of circle and related terminology.
Circle central angles and related problems.
Circle circumference and related problems.
Circle area and related problems.
Triangle area and related problems.
Circle tangents, half-tangents, and related problems.
Volumes of prisms and non-prisms.
Density and weight and related problems.
More-complicated applied problems in trigonometry.

(8) FURTHER TRIGONOMETRIC TOPICS

Definitions of cosecant, secant, and cotangent ratios.
Reciprocal identities, ratio identities, and Pythagorean identities.
Inverse trigonometric notation: arcsin, arccos, arctan.
Measures of rotational speed: angular velocity and circular velocity.
Subdivisions of a degree: decimal, minutes, seconds.

SECTION V: LOGARITHMS

The content of this section consists of common logarithms, natural logarithms, and base "e" exponentials. The content covers the meaning of a logarithm, tables of logarithms, laws of logarithms, calculations by logarithms, and evaluation and rearrangement of logarithmic and exponential formulas.

(1) LOGARITHMS I: INTRODUCTION TO LOGARITHMS

Review of laws of exponents.
Meaning of fractional and decimal exponents.
Validity of laws of exponents for fractional and decimal exponents.
Power-of-ten form of numbers greater than 1.
Use of table of common logarithms.
Definitions of logarithm, characteristic, and mantissa.
Conversion of power-of-ten equations to logarithmic equations, and vice versa.
Use of logarithms to multiply and divide numbers, and to find powers and roots of numbers.
Power-of-ten form of numbers lying between 0 and 1.
Logarithms of numbers lying between 0 and 1.
Laws of logarithms for multiplication, and division, and powers and roots.

(2) LOGARITHMS II: COMMON AND NATURAL LOGARITHMS

Review of positive and negative common logarithms.
Evaluation of logarithmic formulas.
Layout of logarithmic scales.
Reading semi-log graphs and log-log graphs.
Finding common logarithms on the slide rule.
Meaning of base "e" exponentials.
Use of table e^x and e^{-x}.
Evaluation of formulas containing base "e" exponentials.
Meaning of natural logarithms.
Use of table of natural logarithms.
Evaluation of formulas containing natural logarithms.
Graphing exponential equations.

(3) LOGARITHMS III: LAWS AND FORMULAS

Evaluation of more-complicated formulas containing common logarithms or natural logarithms.
Review of the laws of logarithms (common logarithms).
Definition of the logarithmic axiom for equations.
Evaluation of exponential formulas by means of the log axiom.
Rearranging logarithmic formulas by means of the log axiom.
Validity of laws of logarithms for natural logarithms.
Rearranging logarithmic formulas involving natural logarithms.
Rearranging base "e" exponential formulas.
Converting logarithmic formulas to exponential form, and vice versa.

APPENDIX E

SAMPLE COPIES OF POST-TESTS AND DAILY CRITERION TESTS

E-1 Copy of Post-Test (Form C)
For Algebra IX: Formula Rearrangement

E-2 Copies of Daily Criterion Tests #1, #2, #3, and #4
For Algebra IX: Formula Rearrangement

E-3 Copy of Post-Test (Form B)
For Logarithms II: Common and Natural Logarithms

E-4 Copies of Daily Criterion Tests #1, #2, #3, #4, #5, and #6
For Logarithms II: Common and Natural Logarithms

POST-TEST: ALGEBRA IX (Form C)
FORMULA REARRANGEMENT

Directions: Solve each formula for the indicated letter. **Show your work.**

1. Solve for K: $\boxed{F = KS}$

2. Solve for F_2: $\boxed{F_1d_1 = F_2d_2}$

3. Solve for M: $\boxed{H = MS(t_2 - t_1)}$

4. Solve for h: $\boxed{A = \frac{1}{2}h(b_1 + b_2)}$

5. Solve for P: $\boxed{t = \frac{W}{P}}$

6. Solve for M: $\boxed{F = \frac{GMm}{d^2}}$

7. Solve for V_1: $\boxed{\frac{P_1}{P_2} = \frac{V_2}{V_1}}$

8. Solve for K: $\boxed{H = \frac{AKT(t_2 - t_1)}{L}}$

9. Solve for F_1: $\boxed{F_t = F_1 + F_2}$

10. Solve for G: $\boxed{M = K - G}$

1. K =
2. F_2 =
3. M =
4. h =
5. P =
6. M =
7. V_1 =
8. K =
9. F_1 =
10. G =

11. Solve for d_3: $\boxed{d_1F + d_2G + d_3H = 0}$

12. Solve for a: $\boxed{V_1 = V_2 - at}$

11. $d_3 =$

12. $a =$

13. Solve for W: $\boxed{P = T + W(h - h_1)}$

14. Solve for V_1: $\boxed{W = P(V_1 - V_2)}$

13. $W =$

14. $V_1 =$

15. Solve for S: $\boxed{C(T - S) = Q}$

16. Solve for M: $\boxed{G = \frac{L - M}{P}}$

15. $S =$

16. $M =$

17. Solve for f: $\boxed{X_c = \frac{1}{2\pi fC}}$

18. Solve for C: $\boxed{T_1 = \frac{Q}{C} + T_2}$

17. $f =$

18. $C =$

POST-TEST: ALGEBRA IX (Form C)

19. Solve for I: $\boxed{E = IR + Ir}$

20. Solve for A: $\boxed{B = \frac{A}{1 - A}}$

19. $I =$

20. $A =$

21. Solve for R: $\boxed{W = \frac{2Rf}{R - r}}$

22. Solve for H: $\boxed{\frac{1}{F} = \frac{1}{G} - \frac{1}{H}}$

21. $R =$

22. $H =$

23. Solve for V_1: $\boxed{K = \frac{P(V_1 - V_2)}{V_1 T}}$

24. Solve for R_2: $\boxed{R_t = \frac{R_1 R_2}{R_1 + R_2}}$

23. $V_1 =$

24. $R_2 =$

25. Solve for M: $\boxed{E = \frac{M - K}{M}}$

25. $M =$

Daily Test #1: ALGEBRA IX (pp. 1-32)
FORMULA REARRANGEMENT
"Variables; Terms; Rearranging Non-Fractional Formulas Containing Two Terms"

$$\boxed{h = t + \frac{\pi w^2 - 2w}{w_1}}$$

1. How many terms are there on the right side?

2. List the variables which are present on the right side.

1.

2.

3. Solve for r: $\boxed{T = rt}$

4. Solve for V_1: $\boxed{P_1V_1 = P_2V_2}$

3. r =

4. V_1 =

5. Solve for a: $\boxed{bv^2 = aBv_1}$

6. Solve for R: $\boxed{W = kI^2Rt}$

5. a =

6. R =

7. Solve for H:

$$\boxed{2H(D + d) = dh}$$

8. Solve for w_2:

$$\boxed{w_1(t_2 - t_1) = w_2(t_1 + t_2)}$$

7. H =

8. w_2 =

9. Using slide rule, rewrite the right side of the following equation to eliminate the decimal number 6.28 in the denominator:

$$\boxed{x = \frac{1}{6.28fc}}$$

10. Solve for s:

$$\boxed{F = 0.0172s}$$

Write the result in two different forms.

9. x =

10. s =

s =

Daily Test #2: ALGEBRA IX (pp. 33-61)
FORMULA REARRANGEMENT
"Rearranging Formulas Containing Fractions; Addition Axiom; Oppositing Principle"

1. Solve for G: $\boxed{-G = B - A}$

2. Solve for f_2: $\boxed{F - f_1 = -f_2}$

1. G =

2. f_2 =

3. To eliminate "R" from the right side of $\boxed{r = R - P}$, what should you do?

4. To eliminate "A" from the right side of $\boxed{w = A(-d)}$, what should you do?

3.

4.

5. Solve for t: $\boxed{P = \frac{Rd}{t}}$

6. Solve for D: $\boxed{\frac{d}{D} = \frac{F}{f}}$

5. t =

6. D =

7. Solve for r_2: $\boxed{G = \frac{1}{2}(r_1 - r_2)}$

8. Solve for h_2: $\boxed{a_1h_1 + a_2h_2 = 0}$

7. r_2 =

8. h_2 =

9. Solve for W: $\boxed{B = \frac{W - b}{a}}$

10. Solve for R: $\boxed{P = c_1E - c_2R}$

9. W =

10. R =

Daily Test #3: ALGEBRA IX (pp. 62-76)
FORMULA REARRANGEMENT
"Formulas Involving Distributive Principle; Alternate Forms of Solutions"

In Problems 1 and 2, rewrite the right side of each formula in an equivalent form:

1. $V = \dfrac{LH - AKt_1T}{AKT}$

2. $h = C + \dfrac{W}{R}$

1. $V =$

2. $h =$

3. Solve for P: $\boxed{W = h_1P + h_2P}$

4. Solve for M: $\boxed{GM = R + MT}$

3. $P =$

4. $M =$

5. Solve for d_2: $\boxed{W = \frac{1}{2}P(d_1 - d_2)}$

6. Solve for t_2: $\boxed{\frac{1}{T} - \frac{1}{t_1} = \frac{1}{t_2}}$

5. $d_2 =$

6. $t_2 =$

7. Solve for G: $\boxed{E = \frac{G}{G + 1}}$

8. Solve for h: $\boxed{H = 2h + a(d + h)}$

7. $G =$

8. $h =$

Daily Test #4: ALGEBRA IX (pp. 77-97)
FORMULA REARRANGEMENT
"Further Problems In Rearranging Formulas"

1. Write the right side of this formula in the preferred way: $\boxed{H = \dfrac{-AG}{R - T}}$

2. Solve for E: $\boxed{r = \dfrac{E - e}{E}}$

3. Solve for D: $\boxed{\dfrac{1}{d} - \dfrac{1}{D} = \dfrac{1}{F}}$

4. Solve for P: $\boxed{h = \dfrac{P}{1 - BP}}$

5. Solve for s: $\boxed{w = hs + a(h + 1)}$

6. Solve for r: $\boxed{v = \dfrac{Rr}{R + r}}$

7. Solve for a: $\boxed{t_1 = \dfrac{d}{a} + t_2}$

8. Solve for d_1: $\boxed{G = \dfrac{P(d_1 - d_2)}{bd_1}}$

1. H =

2. E =

3. D =

4. P =

5. s =

6. r =

7. a =

8. d_1 =

POST-TEST: LOGARITHMS II - COMMON AND NATURAL LOGARITHMS (Form B)

Directions: In working this test, the following three tables are needed: Common Logs, Natural Logs, and Table of e^x and e^{-x}.

Note: Before starting Problem 1 of this test, be sure that you have completed Problems 26 to 30 on the loose final page, Page 4, which your instructor will provide. Problems 26 to 30 involve finding logarithms on your slide rule.

1. Write the following equation in logarithmic form:

$$\boxed{10^{-2.5467} = 0.00284}$$

2. Write the following equation in exponential form:

$$\boxed{\log 58.6 = 1.7679}$$

1.

2.

3. Find G, if:

$G = \log 703{,}000$

4. Find A, if:

$A = \log 0.387$

3. G =

4. A =

5. Find H, if:

$\log H = 1.4619$

6. Find R, if:

$\log R = -1.4660$

5. H =

6. R =

7. Find P, if: $\boxed{P = h \log K}$

$h = 400$
$K = 27.7$

8. Find D, if: $\boxed{D = 20 \log\left(\frac{I_2}{I_1}\right)}$

$I_2 = 60.0$
$I_1 = 20.0$

7. P =

8. D =

9. Find T, if: $\boxed{\frac{a}{d - a} = \log T}$

$a = 200$
$d = 600$

9. T =

4

10. Find the numerical value of:

$e^{5.19}$

11. Find Q, if:

$Q = e^{-1.81}$

10.

11. Q =

12. Write the following equation in logarithmic form:

$e^{0.5306} = 1.70$

13. Write the following equation in exponential form:

$\ln 0.30 = -1.2040$

12.

13.

14. Find the numerical value of:

$\ln 7.38$

15. Find F, if:

$\ln F = 4.3567$

14.

15. F =

16. $\boxed{B = ae^{ks}}$

Find B, if $a = 8.00$
$k = 1.70$
$s = 2.00$

17. $\boxed{A = \frac{e^{v} + e^{-v}}{2}}$

Find A, if $v = 1.60$

16. B =

17. A =

18. $\boxed{R = Ce^{-\frac{t}{r}}}$

Find R, if $C = 300$
$t = 0.800$
$r = 4.00$

19. $\boxed{w = M(1 - e^{-ht})}$

Find w, if $M = 31.8$
$h = 11.5$
$t = 0.200$

18. R =

19. w =

20. $\boxed{F = \frac{P}{\ln P}}$

Find F, if P = 9.30

21. $\boxed{r = c \ln\left(\frac{B}{K}\right)}$

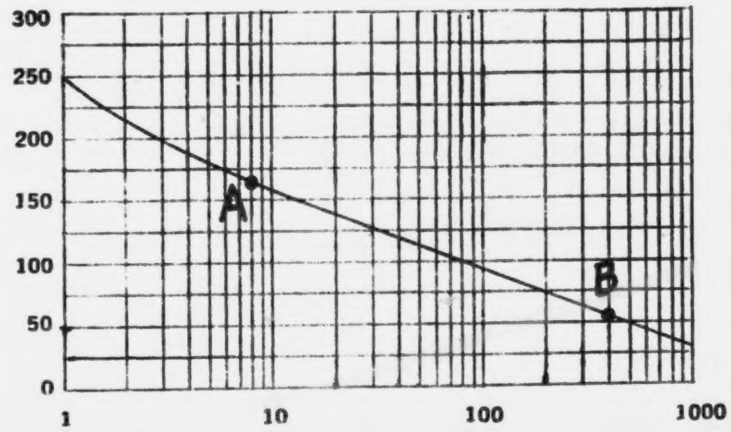
Find r, if c = 40.0
B = 3.76
K = 2.38

20. F =

21. r =

22. List the coordinates of point A.

23. List the coordinates of point B.



22.

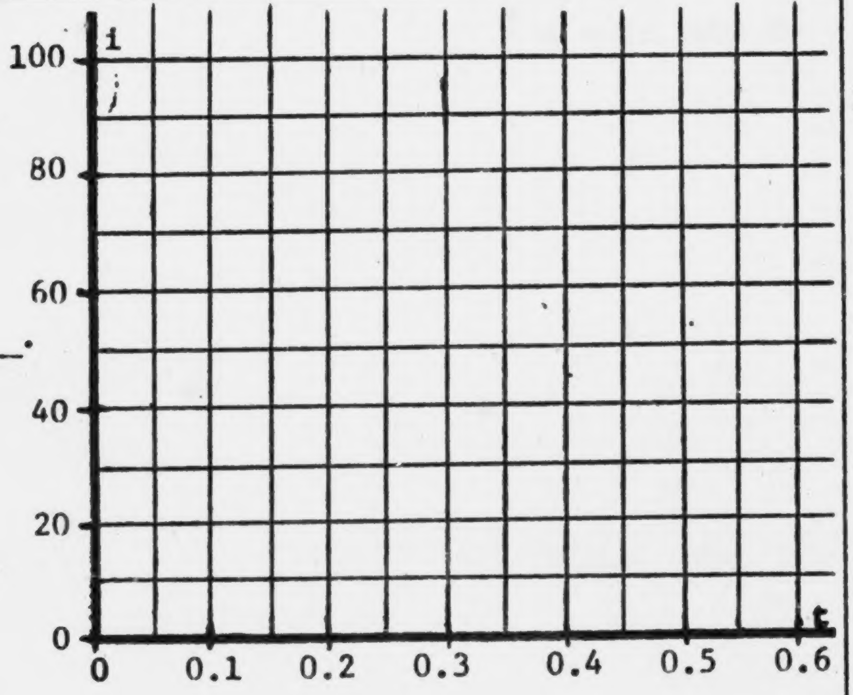
23.

$\boxed{i = 100e^{-4t}}$

24. If t = 0, i = ____.

25. If t = 0.2, i = ____.

26. Graph the equation.



24. i =

25. i =

26. See Graph.

POST-TEST: LOGARITHMS II - COMMON AND NATURAL LOGARITHMS (Form B)

Page 4

Directions: Do the following problems using slide rule only. Do not use a table.

When you have finished these problems, return this sheet to your instructor. He will then give you the first three pages of the test and the three math tables needed.

27. Do this problem on your slide rule:

$\log 2.02 =$ _____

27. ☐

28. Do this problem on your slide rule:

$\log 0.374 =$ _____

28. ☐

29. Do this problem on your slide rule:

If $\log N = 1.734$, then $N =$ _____

29. $N =$

30. Do this problem on your slide rule:

If $\log A = -0.400$, then $A =$ _____

30. $A =$

Daily Test #1: LOGARITHMS II - COMMON AND NATURAL LOGARITHMS (pp. 1-25)
"Review of Common Logarithms"

Where necessary, use the table "Common Logarithms of Numbers."

1. Find the exponent:

   $374.92 = 10^{\square}$

2. Write as a regular number:

   $10^{4.8649} =$ ________

3. List the logarithm:

   $10^{1.9504} = 89.2$

4. List the mantissa:

   $10^{3.7973} = 6{,}270$

5. Write in log form:

   $263 = 10^{2.4200}$

6. Write in exponential form:

   $\log 4.28 = 0.6314$

7. Find h: $\log 16.38 = h$

8. Find T: $\log T = 3.8498$

9. Find the exponent:

   $0.00398 = 10^{\square}$

10. Write as a regular number:

    $10^{-1.7016} =$ ________

11. List the mantissa:

    $\log 0.0061 = -2.2147$

12. List the characteristic:

    $\log 0.5368 = -0.2700$

13. Find H: $\log 0.0001 = H$

14. Find N: $\log N = -2.0000$

15. Find R: $\log R = -0.7408$

16. Find d: $\log 0.0507 = d$

1. $10^{\square}$
2. ☐
3. ☐
4. ☐
5. ☐
6. ☐
7. h =
8. T =
9. $10^{\square}$
10. ☐
11. ☐
12. ☐
13. H =
14. N =
15. R =
16. d =

aily Test #2: LOGARITHMS II - COMMON AND NATURAL LOGARITHMS (pp. 26-38)
"Evaluating 'Log' Formulas; Logarithmic Scale"

here necessary, use the table "Common Logarithms of Numbers."

$\boxed{T = k \log R}$

Find T, if $k = 50$
$R = 7{,}600$

2. $\boxed{N = -\log H}$

Find N, if $H = 0.0309$

$\boxed{D = 20 \log\left(\frac{I_2}{I_1}\right)}$

Find D, if $I_2 = 14.6$
$I_1 = 23.6$

4. $\boxed{G = A - P \log d}$

Find G, if $A = 52.4$
$P = 10.7$
$d = 2{,}880$

5. $\boxed{\log\left(\frac{a + w}{a - w}\right) = s}$

Find s, if $a = 21.7$
$w = 19.3$

6. $\boxed{\frac{F - G}{G} = \log A}$

Find A, if $F = 9.24$
$G = 3.73$

1. T =

2. N =

3. D =

4. G =

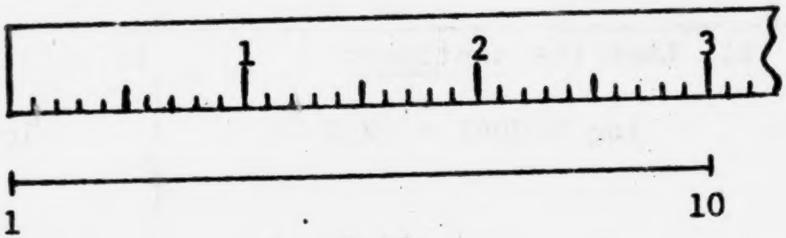
5. s =

6. A =

7. See scale.

7. Calibrate and label a 3-inch log scale at the right. For convenience, part of a ruler calibrated in tenths of an inch is shown.

log 1 = 0.00	log 6 = 0.78
log 2 = 0.30	log 7 = 0.85
log 3 = 0.48	log 8 = 0.90
log 4 = 0.60	log 9 = 0.95
log 5 = 0.70	log 10 = 1.00



Log Scale

8. Find N, if:

$\log(8.16 \times 10^{-5}) = N$

9. Find R, if $\log R = -2.19$

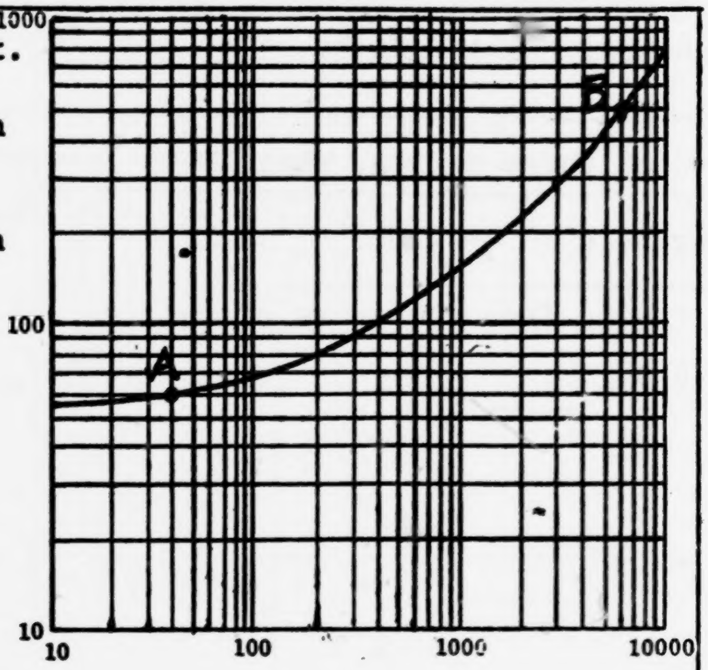
8. N =

9. R =

Daily Test #3: LOGARITHMS II - COMMON AND NATURAL LOGARITHMS (pp. 39-51)
"Semi-Log and Log-Log Graphs; Finding Common Logs on Slide Rule"

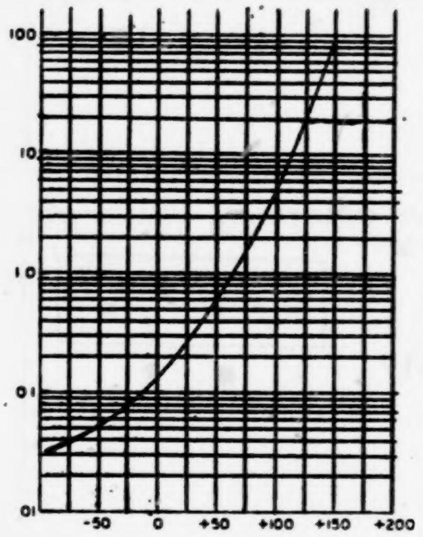
Refer to the graph at the right.

1. How many cycles are shown on the horizontal axis?
2. How many cycles are shown on the vertical axis?
3. Is the graph "semi-log" or "log-log?"
4. List the coordinates of point A.
5. List the coordinates of point B.
6. For a horizontal value of 300, what is the corresponding vertical value?



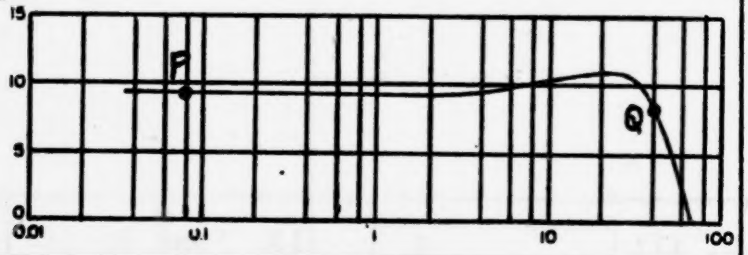
Refer to the graph at the right.

7. Is the graph "semi-log" or "log-log?"
8. Write the coordinates of the point whose abscissa is -75.
9. Write the coordinates of the point whose ordinate is 60.
10. For a horizontal value of 30, what is the corresponding vertical value?



Refer to the graph at the right.

11. Write the coordinates of point P.
12. Write the coordinates of point Q.



Work the following problems on your slide rule. Do not use the logarithm table.

13. log 13,700 = ?

14. log 0.0525 = ?

15. Find N, if log N = 1.895

16. Find A, if log A = -0.582

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
14.
15. N =
16. A =

Daily Test #4: LOGARITHMS II - COMMON AND NATURAL LOGARITHMS (pp. 52-65)
"Base 'e' Exponentials and Tables; Formulas Containing Base 'e' Exponentials"

Where necessary, use the Table of e^x and e^{-x}.

1. Rounded to three digits, what is the numerical value of "e"?

2. Complete: $e^{1.25} = (10^{0.4343})^{1.25} = 10^{0.5429} =$ ___(what number)

Using the table, find the value of each of the following:

3. $e^{6.3} =$ _____

4. $e^{-4.9} =$ _____

5. $e^{0.855} =$ _____

6. $e^{-1.658} =$ _____

7. If $t = 2.38$, $e^{-t} =$ _____

8. If $k = 1.354$, $e^{2k} =$ _____

9. Find the numerical value of: $(e^{3.1})(e^{-3.1}) =$ _____

10. Find W, if: $\boxed{W = \dfrac{e^a + e^{-a}}{2}}$

$a = 0.417$

11. Find N, if: $\boxed{N = Ae^{-hr}}$

$A = 28.2$
$h = 1.54$
$r = 0.720$

12. Find E, if: $\boxed{E = Ke^{-\frac{t}{RC}}}$

$K = 24.3$
$t = 1.63$
$R = 30.4$
$C = 0.0765$

13. Find i, if: $\boxed{i = I_m(1 - e^{-pt})}$

$t = 0.328$
$I_m = 41.7$
$p = 5.85$

1. ☐
2. ☐
3. ☐
4. ☐
5. ☐
6. ☐
7. ☐
8. ☐
9. ☐
10. W =
11. N =
12. E =
13. i =

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Technical Mathematics Project

Daily Test #5: LOGARITHMS II - COMMON AND NATURAL LOGARITHMS (pp. 66-79) "Natural Logarithms and Tables; Formulas Containing Natural Logarithms"

Where necessary, use the Table of Natural Logarithms:

Write each of the following in logarithmic form:

1. $10^{-1.4225} = 0.0378$

2. $e^{1.3350} = 3.80$

Write each of the following in exponential form:

3. $\ln 74.0 = 4.3041$

4. $\log 287 = 2.4579$

5. $\ln 9.40 = ?$

6. $\ln 34.9 = ?$

7. $\ln 0.113 = ?$

8. $\ln\left(\frac{100}{50}\right) = ?$

9. Find R, if: $\ln R = 0.6012$

10. Find H, if: $\ln H = 5.9023$

11. Find P, if: $\ln P = -0.70$

12. Find t, if: $e^t = 6.00$

13. Find N, if: $\boxed{N = K \ln\left(\frac{P}{p}\right)}$

$K = 400$
$P = 0.214$
$p = 0.134$

14. Find G, if: $\boxed{G = \frac{V}{\ln V}}$

$V = 47.0$

15. Find B, if: $\boxed{h + \ln B = 2d}$

$h = 5.20$
$d = 3.80$

16. Find R, if: $\boxed{\log R = d - a}$

$a = 7.50$
$d = 5.90$

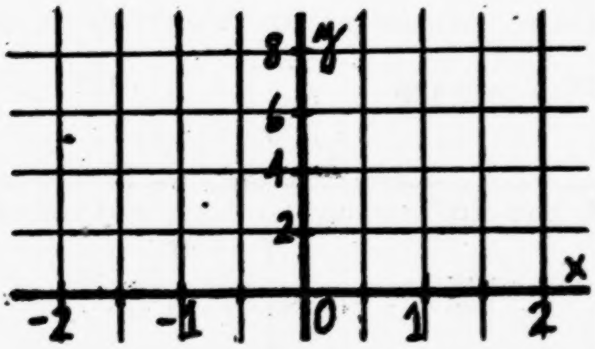
1.
2.
3.
4.
5.
6.
7.
8.
9. R =
10. H =
11. P =
12. t =
13. N =
14. G =
15. B =
16. R =

Daily Test #6: LOGARITHMS II - COMMON AND NATURAL LOGARITHMS (pp. 80-95)
"Graphing Exponential Equations"

Where necessary, use the Table of e^x and e^{-x}.

1. Graph this equation: $\boxed{y = e^x}$

2. Graph this equation: $\boxed{y = e^{-x}}$



1. See graph.

2. See graph.

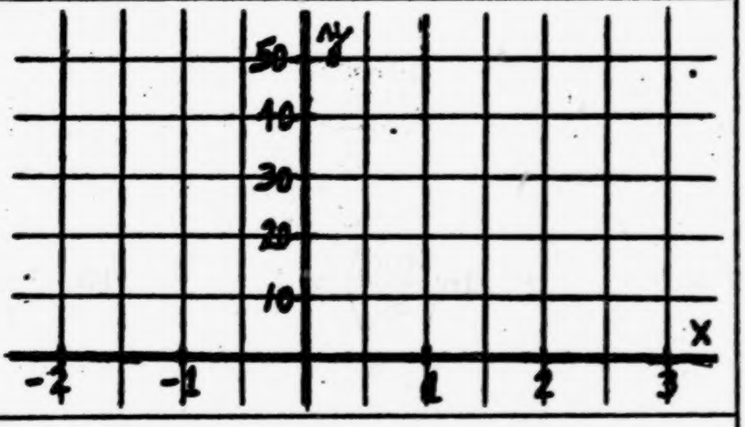
Given: $\boxed{y = 10e^{0.5x}}$

3. If x = 0, y = _____

4. If x = 3, y = _____

5. If x = -1, y = _____

6. Graph the equation.



3. y =

4. y =

5. y =

6. See graph.

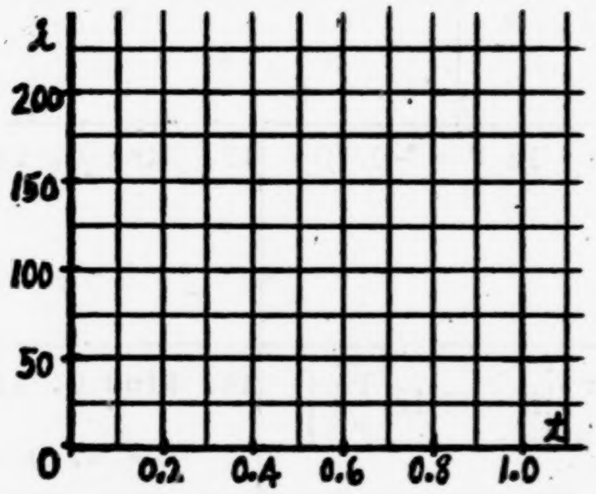
Given: $\boxed{i = 200e^{-3t}}$

7. If t = 0, i = _____

8. If t = 0.1, i = _____

9. If t = 0.5, i = _____

10. Graph the equation.



7. i =

8. i =

9. i =

10. See graph.

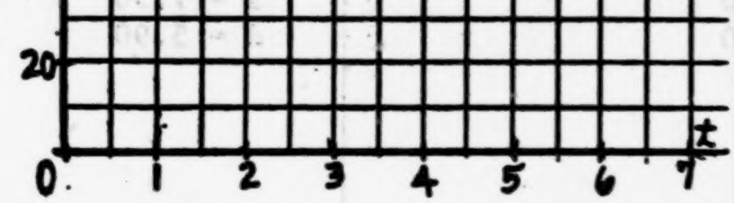
Given: $\boxed{v = 100(1 - e^{-0.4t})}$

11. If t = 0, v = _____

12. If t = 0.5, v = _____

13. If t = 5, v = _____

14. Graph the equation.



11. v =

12. v =

13. v =

14. See graph.

APPENDIX F

COMMON FINAL EXAM IN TECHNICAL MATHEMATICS 1 TAKEN BY PILOT CLASSES AND CONVENTIONAL CLASSES AT MILWAUKEE AREA TECHNICAL COLLEGE (JANUARY, 1966)

F-1 Copy of Common Final Exam in Technical Mathematics 1 (January, 1966)

F-2 Distribution of Scores on Common Final Exam in Technical Mathematics 1 for MATC Pilot Classes and Conventional Classes (January, 1966)

Item Analysis for Common Final Exam in Technical Mathematics 1 for MATC Pilot Classes and Conventional Classes (January, 1966)

FINAL EXAMINATION
MATH 151 TECHNICAL MATHEMATICS 1

Directions: Work each problem in the space provided, and write your answers in the boxes. No separate scratch paper will be permitted. Use your own slide rule. A separate four-place logarithm table will be provided.

Use your time wisely. If you cannot work a problem, omit it, and come back to it later if you have time.

PART I - ALGEBRAIC OPERATIONS

1. Simplify: $6 + (-3) - (+9) - (-5) =$

2. Simplify: $\frac{(-4)(6)(-3)}{(-12)(3)} =$

3. Simplify: $5R - 3(R - 2) - (R + 7) =$

4. Multiply: $(x + 4)(x - 3) =$

5. Factor completely: $6a^2b - 8ab =$

6. Simplify: $\frac{54a^2bc}{9ab^2c} =$

7. Factor completely: $x^2 - 4y^2 =$

8. Add: $\frac{2P}{3} + \frac{P}{2} =$

1.

2.

3.

4.

5.

6.

7.

8.

9. Multiply: $\left(\frac{xy}{3}\right)\left(\frac{6x}{y}\right) =$	9.
10. Simplify: $\dfrac{\frac{a}{2b}}{\frac{b}{2a}} =$	10.

PART II - EXPONENTS AND RADICALS

11. Simplify: $\dfrac{(10^2)(10^{-3})}{10^{-4}} =$	11.
12. Simplify: $\left(\sqrt{3RT}\right)^2 =$	12.
13. Simplify: $\sqrt{a^2b} \cdot \sqrt{4b} =$	13.
14. Find the numerical value of: $\sqrt{3^2 + 4^2}$	14.
15. Simplify: $(b^{2.5})^{0.4} =$	15.
16. Change to radical form: $(N)^{\frac{2}{3}} =$	16.
17. Change to radical form: $(3K)^{0.2} =$	17.
18. Change to exponent form: $\sqrt[5]{W^2} =$	18.

PART III - SIMPLE EQUATIONS

19. Solve for x: $\frac{x}{5} = \frac{16}{20}$	19. $x =$

20. Solve for W: $5 - W = 8$

20. W =

21. Solve for y: $2y + 6 = 6 - 3y$

21. y =

22. Solve for x: $x - 2 = \frac{x}{2} + 3$

22. x =

23. Solve for R: $\frac{4}{R} + 5 = \frac{3}{2R}$

23. R =

24. Solve for x: $x - \frac{3x - 2}{2} = 5$

24. x =

PART IV - FORMULA REARRANGEMENT

25. Solve for L: $A = LW$

25. L =

26. Solve for H: $A = \frac{1}{2}BH$

26. H =

27. Solve for F_2: $F_t = F_1 - F_2$

27. F_2 =

28. Solve for V_2: $\frac{P_2}{P_1} = \frac{V_1}{V_2}$

28. $V_2 =$

29. Solve for i: $e = E - iR$

29. $i =$

30. Solve for w: $2P = \frac{m + n}{w}$

30. $w =$

31. Solve for S: $M = A(R - S)$

31. $S =$

32. Solve for A: $B = \frac{A}{1 - A}$

32. $A =$

33. Solve for Q: $\frac{1}{P} + \frac{1}{Q} = \frac{1}{R}$

33. $Q =$

PART V: SLIDE RULE OPERATIONS

Work these problems on your slide rule:

34. $8.45 \times 2.36 =$

34.

35. $\frac{735}{0.364} =$

35.

36. $\sqrt{2250.} =$

36.

37. $(1.45 \times 2.47)^2 =$

37.

38. $\frac{4.44 \times 37.6}{6.05} =$

38.

39. In calculating $\frac{0.0328 \times 76{,}600.}{0.517}$ on the slide rule, the correct sequence "486" was obtained. With decimal point correctly placed, the actual answer is:

39.

PART VI - SYSTEMS OF EQUATIONS

40. Solve this system for x and y:

$$\begin{aligned} x + 2y &= 3 \\ x - y &= 6 \end{aligned}$$

40. x =

y =

41. Solve this system for G and H:

$$\begin{aligned} G + H &= 12 \\ 0.2G + 0.5H &= 3 \end{aligned}$$

41. G =

H =

42. Eliminate I from these equations and solve for R:

$P = EI$
$E = IR$

42. R =

PART VII - EXPONENTIALS AND LOGARITHMS

Use the special log table where needed in this section. The word log means base 10 logarithms, and ln means base e logarithms.

43. Write in logarithmic form: $10^{2.3010} = 200$	43.
44. Write in exponential form: log 62.8 = 1.7980	44.
45. log 87,200 =	45.
46. log 0.00307 =	46.
47. If log N = 3.8615, then N =	47. N =
48. If $10^t = 31.5$, then find the numerical value of t.	48. t =
49. If $R = 10^{3.8645}$, then find the numerical value of R.	49. R =
50. Given: $P = 51.3 \times 7.95$, and log 51.3 = 1.7101 log 7.95 = 0.9004 Find: a. log P = b. P =	50. a. log P = b. P =
51. Given: $R = (3350.)^{0.1}$, and log 3350. = 3.5250 Find: a. log R = b. R =	51. a. log R = b. R =

52. Find x: $(100)^x = 1000$

52. x =

53. Given this formula: D = 20(log P)
If D = 60, find P.

53. P =

54. Write in logarithmic form: $e^{-2.6} = 0.0743$
(where e = 2.72)

54.

55. Write in exponential form: ln 9.70 = 2.2721

55.

56. Given: log e = 0.4343

If $M = e^{10}$, find M.

56. M =

57. See graph.

58. See graph.

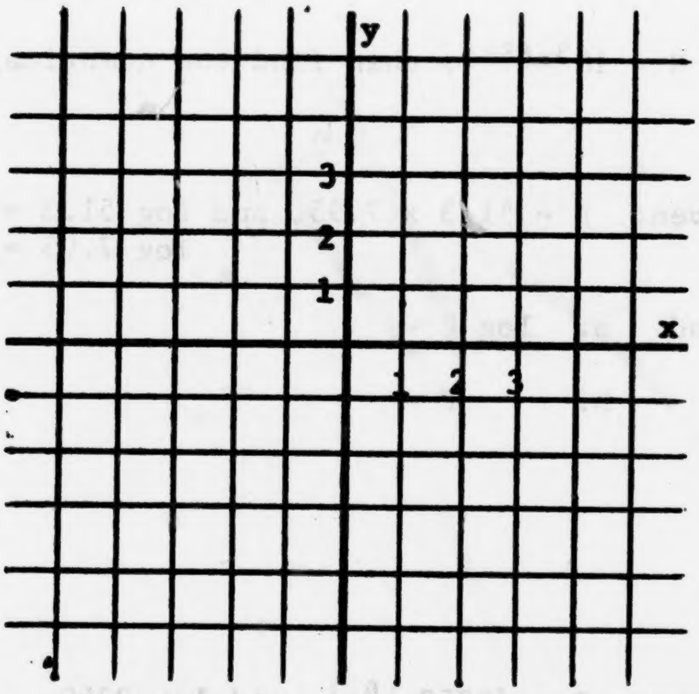
PART VIII - LINEAR EQUATIONS AND GRAPHING

57. On the coordinate system at the right, graph this equation:

$x + y = 5$

58. On the coordinate system at the right, graph this equation:

$2x - y = 4$



59. From the graph, determine the coordinates of the intersection point of the two lines:

59. (,)

60. Find the slope "m" and the y-intercept "b" of the line whose equation is: $3x - y = 7$

60. $m =$

$b =$

61. Find the equation of the line of slope $m = 3$ which passes through the point $(1,-2)$.

61.

62. A line passes through the points $(2,-1)$ and $(-1,5)$.

a. Find its slope.

62. a. $m =$

b. Find its equation.

b.

PART IX - QUADRATIC AND RADICAL EQUATIONS

63. The quadratic formula is the solution of the quadratic equation $ax^2 + bx + c = 0$. List the quadratic formula.

63. $x =$

64. Solve for x: $3x^2 - 5 = 2x^2 + 20$

64.

65. Solve for x: $5x = x^2 - 24$

65.

66. Solve for x: $x^2 - 4x + 5 = 0$

66.

67. Solve for x: $\sqrt{2x + 8} = \sqrt{4x}$

67.

68. Solve for x: $\sqrt{x} - 2 = 1$

68.

69. Solve for x: $\sqrt{x + 5} = 1 - x$

69.

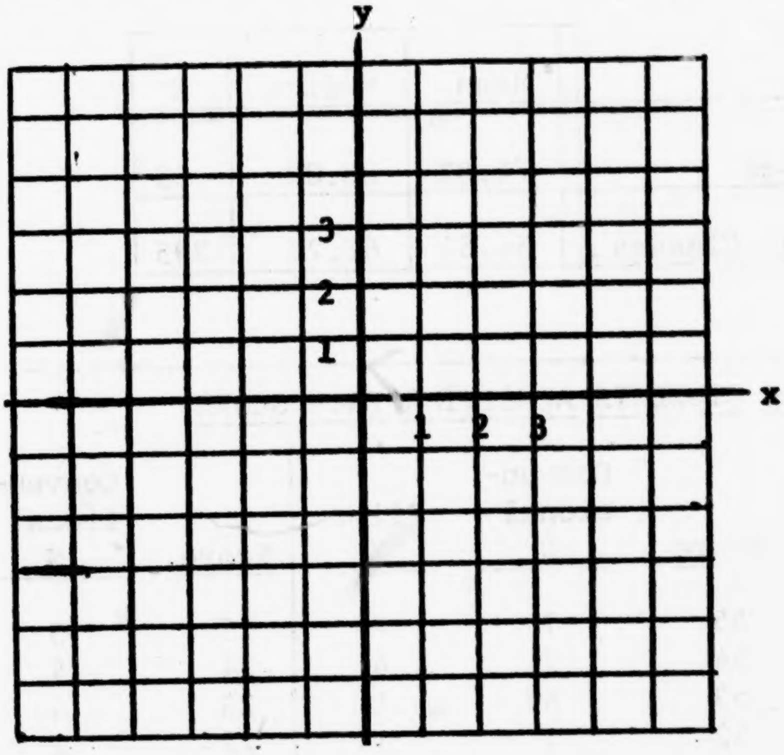
70. Rearrange this formula, solving it for H:

$R = \sqrt{2GH}$

70. H =

71. On the coordinate system below, sketch the graph of $y = 4x - x^2$.

71. See graph.



72. Write the coordinates of the turning point (vertex) of:

$y = 4x - x^2$

72. (,)

73. From the graph, determine the roots of:

$4x - x^2 = 0$

73.

DISTRIBUTION OF SCORES ON COMMON FINAL EXAM IN TECHNICAL MATHEMATICS 1 FOR MATC PILOT CLASSES AND CONVENTIONAL CLASSES (JANUARY, 1966)

	Mean	Median	N
Pilot Classes	74.8%	80.0%	59
Conventional Classes	56.5%	61.2%	295

NUMBER OF STUDENTS ACHIEVING EACH SCORE

Score	Conventional N	Pilot N	Score	Conventional N	Pilot N	Score	Conventional N	Pilot N
85 max.	-	2	55	7	-	25	5	-
84	2	1	54	7	4	24	4	-
83	-	-	53	6	1	23	4	-
82	2	1	52	7	1	22	2	-
81	-	2	51	3	1	21	2	-
80	1	2	50	5	-	20	1	-
79	2	2	49	3	1	19	3	-
78	4	1	48	7	-	18	1	-
77	4	3	47	6	1	17	1	-
76	1	1	46	1	1	16	4	-
75	3	-	45	5	-	15	1	-
74	1	1	44	2	2	14	2	-
73	3	2	43	5	1	13	1	-
72	11	2	42	5	1	12	3	-
71	5	1	41	8	-	11	2	-
70	5	2	40	5	1	10	3	-
69	4	3	39	3	-	9	2	-
68	6	4	38	6	-	8	2	-
67	5	1	37	3	-	7	1	-
66	4	1	36	1	1	6	2	-
65	5	-	35	5	-	5	1	-
64	9	1	34	3	-	4	2	-
63	3	1	33	2	1	3	-	-
62	6	-	32	1	-	2	1	-
61	11	1	31	4	1	1	-	-
						0	-	-
60	3	2	30	1	-			
59	4	-	29	2	-		N = 295	N = 59
58	6	1	28	5	-			
57	3	2	27	4	-			
56	8	1	26	2	-			

ITEM ANALYSIS FOR COMMON FINAL EXAM IN TECHNICAL MATHEMATICS 1 FOR MATC PILOT CLASSES AND CONVENTIONAL CLASSES (JANUARY, 1966)

Exam Item	Conventional	Pilot	Pilot Gains
Part I - Algebraic Operations			
1	82%	90%	+ 8%
2	71%	73%	+ 2%
3	58%	63%	+ 5%
4	85%	100%	+15%
5	77%	92%	+15%
6	74%	73%	- 1%
7	83%	75%	- 8%
8	74%	73%	- 1%
9	53%	64%	+11%
10	47%	53%	+ 6%
Part II - Exponents and Radicals			
11	53%	71%	+18%
12	85%	92%	+ 7%
13	55%	53%	- 2%
14	73%	83%	+10%
15	58%	73%	+15%
16	77%	98%	+21%
17	30%	73%	+43%
18	79%	100%	+21%
Part III - Simple Equations			
19	94%	98%	+ 4%
20	87%	97%	+10%
21	81%	93%	+12%
22	75%	86%	+11%
23	45%	68%	+23%
24	25%	31%	+ 6%
Part IV - Formula Rearrangement			
25	87%	98%	+11%
26	74%	100%	+26%
27	75%	93%	+18%
28	69%	98%	+29%
29	62%	85%	+23%
30	76%	98%	+22%
31	56%	85%	+29%
32	36%	78%	+42%
33	38%	75%	+37%
Part V - Slide Rule Operations			
34	63%	92%	+29%
35	28%	58%	+30%
36	25%	59%	+34%
37	48%	76%	+28%
38	48%	80%	+32%
39	45%	68%	+23%

Exam Item	Conventional	Pilot	Pilot Gains
Part VI - Systems of Equations			
40 a.	78%	92%	+14%
b.	75%	93%	+18%
41 a.	54%	68%	+14%
b.	52%	66%	+14%
42	33%	86%	+53%
Part VII - Exponentials and Logarithms			
43	79%	86%	+ 7%
44	80%	85%	+ 5%
45	71%	86%	+15%
46	64%	59%	- 5%
47	70%	90%	+20%
48	64%	92%	+28%
49	66%	95%	+29%
50 a.	76%	88%	+12%
b.	56%	90%	+34%
51 a.	44%	78%	+34%
b.	35%	76%	+41%
52	26%	83%	+57%
53	19%	36%	+17%
54	46%	59%	+13%
55	46%	80%	+34%
56	16%	31%	+15%
Part VIII - Linear Equations and Graphing			
57	82%	83%	+ 1%
58	67%	73%	+ 6%
59	65%	78%	+13%
60 a.	56%	61%	+ 5%
b.	52%	66%	+14%
61	29%	25%	- 4%
62 a.	39%	39%	0%
b.	22%	22%	0%
Part IX - Quadratic and Radical Equations			
63	62%	88%	+26%
64 a.	75%	90%	+15%
b.	47%	78%	+31%
65 a.	62%	76%	+14%
b.	59%	75%	+16%
66 a.	12%	46%	+34%
b.	11%	46%	+35%
67	64%	88%	+24%
68	56%	75%	+19%
69 a.	34%	46%	+12%
b.	21%	29%	+ 8%
70	60%	83%	+23%
71 a.	45%	64%	+19%
b.	48%	71%	+23%
72	53%	81%	+28%
73 a.	42%	69%	+27%
b.	42%	69%	+27%

APPENDIX G

DATA FOR FINAL EXAMINATION IN TECHNICAL MATHEMATICS 1 (JANUARY, 1969)

G-1 Copy of Final Exam in Technical Mathematics 1
(January, 1969)

G-2 Distribution of Scores for Final Examination
in Technical Mathematics 1 - January, 1969
Milwaukee Area Technical College

Item Analysis for Final Examination
in Technical Mathematics 1 - January, 1969
Milwaukee Area Technical College

Milwaukee Area Technical College
Technical Mathematics Project

FINAL EXAMINATION
MATH 151 TECHNICAL MATHEMATICS 1

Directions: Work each problem in the space provided. Show all necessary work. Do not use separate scratch paper. Write your answers in the boxes. Use slide rule where necessary. Two math tables will be provided, "Logarithms" and "Trig Ratios."

The time for the test is 1 hour and 45 minutes. Do not spend too much time on any single problem. If you cannot work a problem, skip it, and come back to it later.

Part I - Signed Numbers

1. $(-7) + (-4) = ?$

2. $-3 - (-7) = ?$

3. $(-4) - 9 = ?$

4. $7 + (-3) - (+9) - (-2) = ?$

5. $5(-1)(-4) = ?$

6. $\frac{(-5)(6)(-3)}{(-15)(3)} = ?$

1.
2.
3.
4.
5.
6.

Part II - Elementary Equations

7. Solve for w:

$20 = 35 - 3w$

8. Solve for H:

$5 - H = 8$

9. Solve for y:

$2y + 7 = 7 - 3y$

10. Solve for r:

$3r + 2(r + 8) = -4$

7. w =
8. H =
9. y =
10. r =

11. Solve for t:

$$2t - (3 - t) = 3$$

12. Solve for x:

$$5 - (3 + 2x) = 4 - 2(3 - x)$$

11. t =

12. x =

Part III - Fractional Equations

13. Solve for s:

$$\frac{2}{3s} = 5$$

14. Solve for h:

$$\frac{h}{5} = \frac{8}{20}$$

13. s =

14. h =

15. Solve for d:

$$d - 3 = \frac{d}{2} + 2$$

16. Solve for y:

$$\frac{2}{y} + 5 = \frac{1}{3y}$$

15. d =

16. y =

17. Solve for w:

$$\frac{3}{w} - \frac{2}{3} = \frac{5}{2w}$$

18. Solve for b:

$$b - \frac{3b - 4}{2} = 3$$

17. w =

18. b =

Part IV - Formula Rearrangement

19. Solve for G:

$$P = \frac{A}{G}$$

20. Solve for a:

$$P = \frac{1}{2}a(b + d)$$

19. G =

20. a =

21. Solve for F_3:

$$F_1 = F_2 - F_3$$

22. Solve for R_2:

$$\frac{H_2}{H_1} = \frac{R_1}{R_2}$$

21. F_3 =

22. R_2 =

23. Solve for h:

$$r = R - ht$$

24. Solve for w:

$$2E = \frac{p + r}{w}$$

23. h =

24. w =

25. Solve for P:

$$A = F(T - P)$$

26. Solve for E:

$$G = \frac{E}{1 - E}$$

25. P =

26. E =

27. Solve for H:

$$\frac{1}{B} + \frac{1}{H} = \frac{1}{R}$$

28. Solve for W:

$$T = \frac{R}{W} + F$$

27. H =

28. W =

Part V - Powers of Ten

29. $\dfrac{10^2 \times 10^{-3}}{10^{-4}} = 10^?$

30. $\dfrac{10^{-2} \times 10^5}{10^6 \times 10^{-3}} = 10^?$

31. Write 0.379 in standard notation:

$0.379 = \underline{\ ?\ } \times 10^?$

32. Write in regular number form:

$61.4 \times 10^{-3} = \underline{\ ?\ }$

33. $0.0142 \times 10^{-2} = \underline{\ ?\ } \times 10^{-6}$

34. $0.00637 \times 10^{-1} = 6.37 \times 10^?$

29. $10^{\boxed{}}$

30. $10^{\boxed{}}$

31. $\underline{\qquad} \times 10^{\boxed{}}$

32.

33.

34. $10^{\boxed{}}$

Part VI - Estimation

Estimate the answer for each problem. Then use the estimate to place the decimal point in the digits of the slide rule answer (shown in quotation marks).

35. $0.000775 \times 41{,}600$ "322"

36. $\dfrac{0.0462}{58.8}$ "785"

37. $\dfrac{0.0324 \times 66{,}500}{0.472}$ "456"

38. $\dfrac{0.0713}{0.00528 \times 3{,}210}$ "421"

39. $\sqrt{0.0000684}$ "827"

40. $\sqrt[3]{317{,}000}$ "682"

35.

36.

37.

38.

39.

40.

Part VII - Slide Rule Operations

Work these problems on your slide rule. Do NOT use long arithmetic methods.

41. $7.35 \times 2.26 = ?$

42. $\frac{595}{0.286} = ?$

43. $\frac{3.44 \times 35.8}{5.05} = ?$

44. $\frac{92.5}{2.22 \times 32.8} = ?$

45. $\sqrt{2150} = ?$

46. $(2.83 \times 1.35)^2 = ?$

41.

42.

43.

44.

45.

46.

Part VIII - Algebraic Fractions

Answers must be in lowest terms.

47. Multiply: $\left(\frac{2t}{d}\right)\left(\frac{3r}{4t}\right)$

48. Divide: $\frac{\frac{hr}{t}}{r}$

49. Complete: $\frac{d}{6r} = \left(\frac{d}{2}\right)(?)$

50. Add: $\frac{x}{2} + \frac{x}{3}$

51. Subtract: $\frac{b}{a} - \frac{r}{t}$

52. Simplify: $\frac{a}{1 + \frac{1}{r}}$

47.

48.

49.

50.

51.

52.

Part IX - Graphing

53. In the equation $\boxed{3x - y = 10}$ if $x = 1$, $y = ?$

54. In the equation $\boxed{EI = 20}$ if $I = 20$, $E = ?$

53. y =

54. E =

Write the coordinates of:

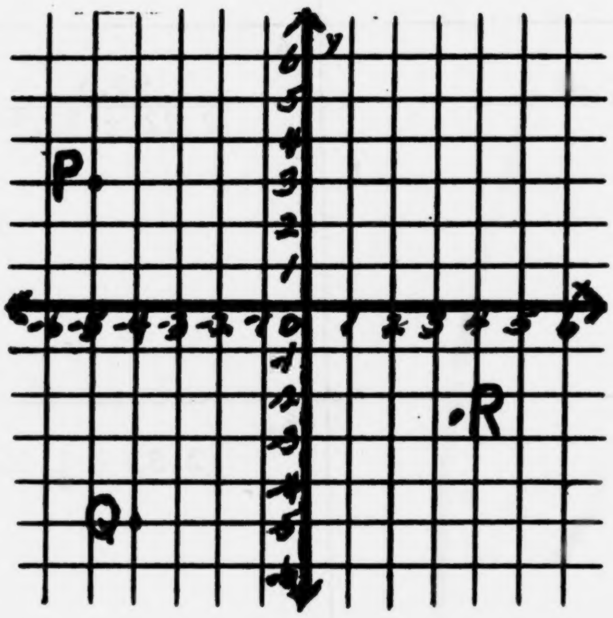
55. Point P

56. Point Q

57. Point R

58. Point R lies in what quadrant?

59. What is the abscissa of point Q?



55.

56.

57.

58.

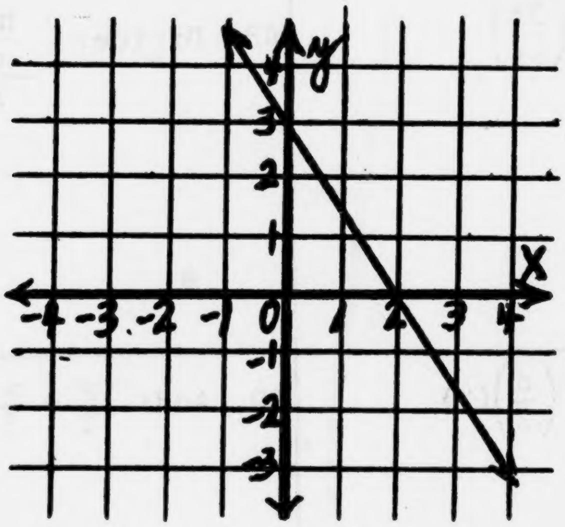
59.

Refer to the straight line graph shown below, and answer the following:

60. Write the coordinates of the x-intercept of the graph.

61. Write the coordinates of the y-intercept of the graph.

62. On the axes at the right, construct the graph of: $\boxed{y = x + 3}$



60.

61.

62. See graph at left.

Part X - Logarithms

63. Change to exponent form:

$$\sqrt[4]{P^3}$$

64. Change to radical form:

$$G^{\frac{2}{5}}$$

63.

64.

65. Using the table, write this power of ten as a regular number:

$10^{1.5756} = ?$

66. Using the table, write 6,180 in power-of-ten form:

$6,180 = 10^?$

65.

66. 10

67. log 28,300 = ?

68. log 0.0402 = ?

67.

68.

69. Find N (in regular number form) if:

log N = 2.8149

70. Write this exponential equation in logarithmic form:

$10^{2.5587} = 362$

69. N =

70.

71. Using logarithms, find the numerical value of P, if:

$P = 18.2 \times 4.18$

Given: log 18.2 = 1.2601
log 4.18 = 0.6212

72. Using logarithms, find the numerical value of R, if:

$R = (5,180)^{0.1}$

Given: log 5,180 = 3.7143

71. P =

72. R =

(No credit will be given unless your work is shown above.)

Part XI - Trigonometry

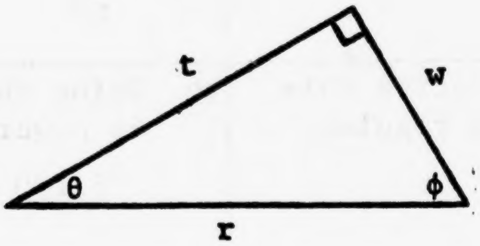
Where necessary, use the "Trigonometric Ratios" table which has been provided.

Using the right triangle shown, define these trigonometric ratios of angle θ and angle φ:

73. cos θ = ?

74. sin θ = ?

75. tan φ = ?
(Note: The angle is φ)

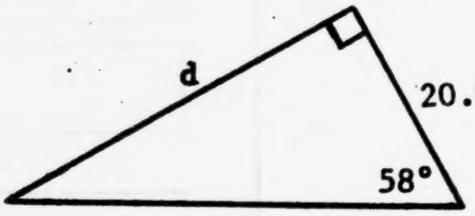


73. cos θ =

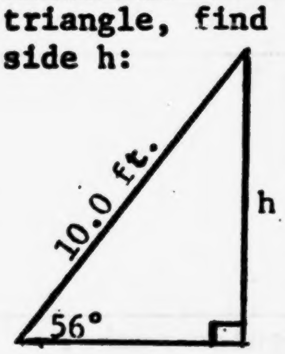
74. sin θ =

75. tan φ =

76. In this right triangle, find the length of side d:



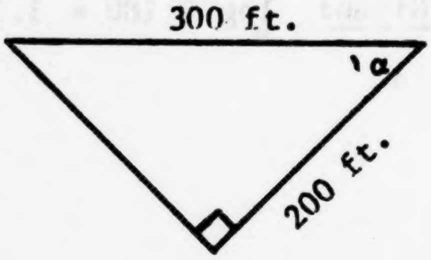
77. In this right triangle, find the length of side h:



76. d = in.

77. h = ft.

78. In this right triangle, find the size of angle α, to the nearest degree.



78. α =

DISTRIBUTION OF SCORES FOR FINAL EXAMINATION IN TECHNICAL MATHEMATICS 1 - JANUARY, 1969 MILWAUKEE AREA TECHNICAL COLLEGE

Mean = 82.9%
Median = 87.2%
N = 336

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
78	4	1.2%	1.2%
77	10	3.0%	4.2%
76	14	4.2%	8.4%
75	21	6.2%	14.6%
74	19	5.6%	20.2%
73	25	7.4%	27.6%
72	22	6.5%	34.1%
71	20	6.0%	40.1%
70	10	3.0%	43.1%
69	12	3.6%	46.7%
68	13	3.9%	50.6%
67	11	3.3%	53.9%
66	13	3.9%	57.8%
65	8	2.4%	60.2%
64	7	2.1%	62.3%
63	15	4.4%	66.7%
62	13	3.9%	70.6%
61	6	1.7%	72.3%
60	4	1.2%	73.5%
59	10	3.0%	76.5%
58	5	1.5%	78.0%
57	5	1.5%	79.5%
56	8	2.4%	81.9%
55	3	0.9%	82.8%
54	6	1.7%	84.5%
53	5	1.5%	86.0%
52	4	1.2%	87.2%
51	6	1.7%	88.9%
50	5	1.5%	90.4%
49	3	0.9%	91.3%
48	5	1.5%	92.8%
47	1	0.3%	93.1%
46	5	1.5%	94.6%
45	4	1.2%	95.8%
44	1	0.3%	96.1%
43	1	0.3%	96.4%
42	1	0.3%	96.7%
41			
40	1	0.3%	97.0%
39			
38	1	0.3%	97.3%
37	1	0.3%	97.6%
36	2	0.6%	98.2%
35			
34			
33			
32			
31	1	0.3%	98.5%
30	1	0.3%	98.8%
29			
28	1	0.3%	99.1%
27	1	0.3%	99.4%
26			
25			
24	1	0.3%	99.7%
7	1	0.3%	100.0%

ITEM ANALYSIS FOR FINAL EXAMINATION IN TECHNICAL MATHEMATICS 1 - JANUARY, 1969
MILWAUKEE AREA TECHNICAL COLLEGE

Mean = 82.9%
Median = 87.2%
N = 336

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	%
Signed Numbers	1.	99%
	2.	99%
	3.	97%
	4.	93%
	5.	94%
	6.	82%
Elementary Equations	7.	95%
	8.	95%
	9.	94%
	10.	96%
	11.	90%
	12.	82%
Fractional Equations	13.	84%
	14.	94%
	15.	82%
	16.	68%
	17.	70%
	18.	37%
Formula Rearrangement	19.	92%
	20.	79%
	21.	85%
	22.	83%
	23.	78%
	24.	90%
	25.	78%
	26.	63%
	27.	75%
	28.	75%
Powers of Ten	29.	83%
	30.	85%
	31.	94%
	32.	96%
	33.	78%
	34.	86%
Estimation	35.	88%
	36.	67%
	37.	68%
	38.	78%
	39.	82%
	40.	85%
Slide Rule Operations	41.	91%
	42.	68%
	43.	85%
	44.	77%
	45.	81%
	46.	80%
Algebraic Fractions	47.	84%
	48.	27%
	49.	97%
	50.	77%
	51.	66%
	52.	46%
Graphing	53.	89%
	54.	69%
	55.	96%
	56.	98%
	57.	94%
	58.	95%
	59.	84%
	60.	93%
	61.	90%
	62.	87%
Logarithms	63.	93%
	64.	89%
	65.	83%
	66.	85%
	67.	71%
	68.	33%
	69.	83%
	70.	74%
	71.	81%
	72.	52%
Trigonometry	73.	95%
	74.	96%
	75.	94%
	76.	86%
	77.	89%
	78.	86%

APPENDIX H

DATA FOR FINAL EXAMINATION IN TECHNICAL MATHEMATICS 2 (MAY, 1969)

H-1 Copy of Final Exam in Technical Mathematics 2
(May, 1969)

H-2 Distribution of Scores for Final Examination
Technical Mathematics 2 - May, 1969
Milwaukee Area Technical College

Item Analysis for Final Examination
Technical Mathematics 2 - May, 1969
Milwaukee Area Technical College

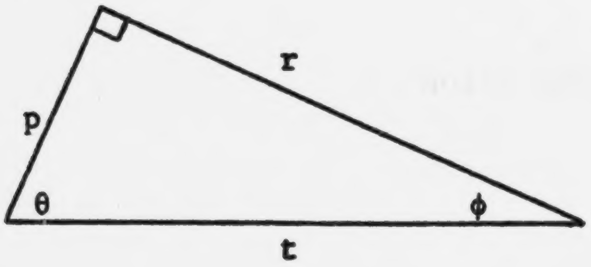
FINAL EXAMINATION
MATH 152 TECHNICAL MATHEMATICS 2

Directions: Work each problem in the space provided. Show all necessary work. Do not use separate scratch paper. Write your answers in the boxes. Use slide rule where necessary. Four math tables will be provided: "Trigonometric Ratios," "Common Logarithms," "Tables of e^x and e^{-x}," and "Natural Logarithms."

The time for the test is 1 hour and 45 minutes. Do not spend too much time on any single problem. If you cannot work a problem, skip it, and come back to it later.

Part I: Trigonometric Ratios

Using the right triangle shown, define the following trigonometric ratios:



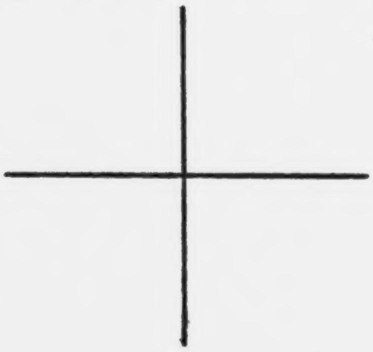
For angle θ:
1. tan θ
2. cos θ
3. csc θ

For angle ϕ:
4. cot ϕ
5. sin ϕ
6. sec ϕ

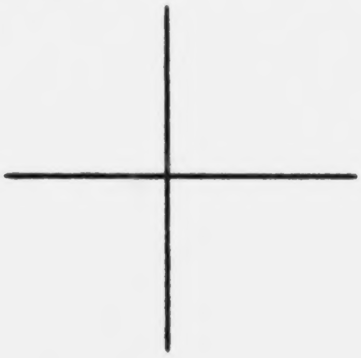
1. tan θ =
2. cos θ =
3. csc θ =
4. cot ϕ =
5. sin ϕ =
6. sec ϕ =

Using a "Trig Ratios" table, find the numerical value of each:

7. cos 160° = ?

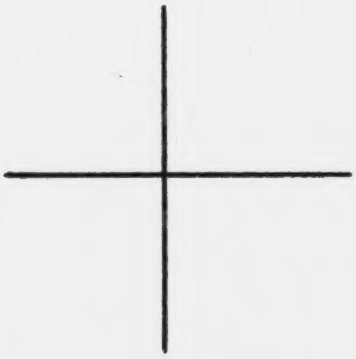


8. tan 220° = ?



9. sin 670° = ?

10. sin (-330°) = ?



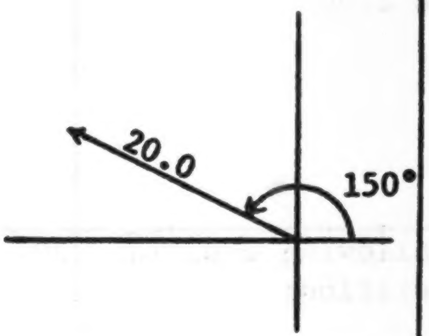
7.
8.
9.
10.

Part II - Vectors

A vector 20.0 units long has a standard position angle of 150°.

11. Find the horizontal component of the vector.

12. Find the vertical component of the vector.

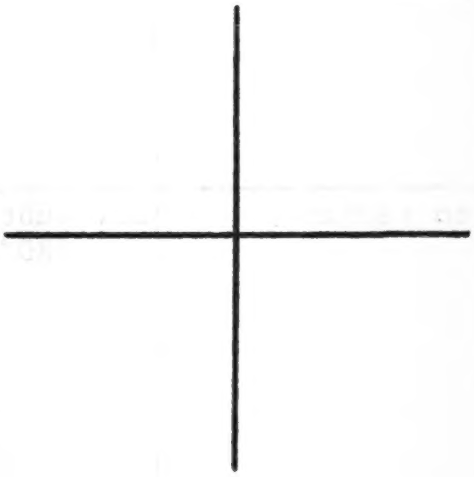


11. units

12. units

A vector has a horizontal component of 20.0 units and a vertical component of -20.0 units.

13. Find the standard position angle of the vector.



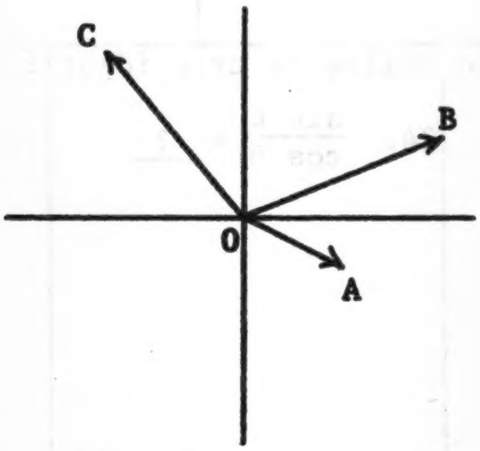
13.

The endpoints of the three vectors at the right are:

$\overrightarrow{OA}$: (2,-1)
$\overrightarrow{OB}$: (4,2)
$\overrightarrow{OC}$: (-3,4)

14. Find the coordinates of the endpoint of their resultant.

15. Find the coordinates of the endpoint of their equilibrant.



14.

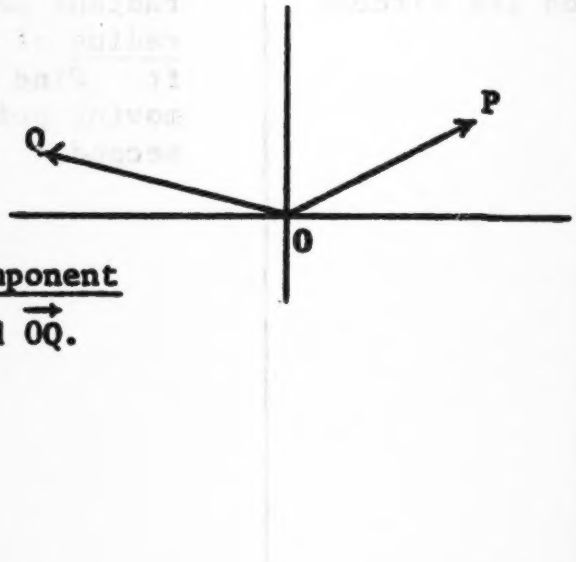
15.

In the diagram at the right:

$\overrightarrow{OP}$ is 80.0 units long, and its standard position angle is 30°.

$\overrightarrow{OQ}$ is 100 units long, and its standard position angle is 160°.

16. Calculate the vertical component of the resultant of $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.



16.

Part III: Inverse Trig Notation; Radians; Identities

17. Find the numerical value of G:

$G = \arctan 2.90$

18. Find the numerical value of N:

$\cos^{-1} N = 34°$

17. G =

18. N =

19. Write the following equation in arcsin notation:

$R = \sin H$

20. Convert 2 radians to degrees.

19.

20.

21. Convert 180° to radians.

22. Subtract 141°15'45" from 180°.

21. rad.

22.

Complete each of the following trig identities:

23. $\frac{1}{\csc \theta} = \underline{\ ?\ }$

24. $\frac{\sin \theta}{\cos \theta} = \underline{\ ?\ }$

25. $\underline{\ ?\ } + \cos^2\theta = 1$

23.

24.

25.

26. A wheel of radius 3.60 in. rotates through an angle of 40°. Through what distance does a point on its circumference move?

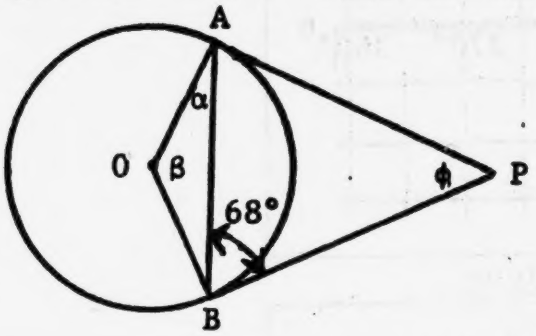
27. A point on the circumference of a rotating circle has an angular velocity of 60.0 radians per second. The radius of the circle is 0.500 ft. Find the velocity of the moving point, in feet per second.

26. in.

27. ft./sec.

Part IV: Geometry and Applied Trig

In the diagram below, AP and BP are tangent to circle O. Angle ABP = 68°. Find the following angles:



28. Angle APB = Angle ϕ = ____

29. Angle OAB = Angle α = ____

30. Angle AOB = Angle β = ____

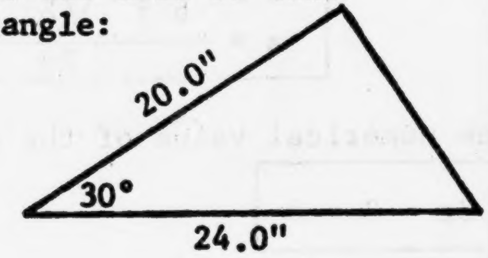
28. ϕ =

29. α =

30. β =

31. The area of a circle is 26.0 square inches. Find the diameter.

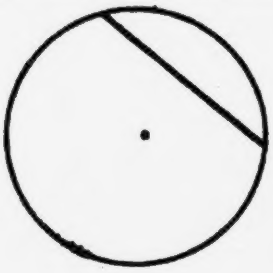
32. Find the area of this triangle:



31. in.

32. sq. in.

33. A circle's diameter is 40.0". The circumference is divided into three equal arcs. A chord is drawn on one of the arcs. Find the length of the chord.



33. in.

Part V: Sine Waves

34. $\boxed{y = 250 \sin(\theta + 90°)}$

Find y when $\theta = 0°$.

35. Write the equation of the following sine wave:

Fifth harmonic whose amplitude is 18.

34. y =

35.

36. A sine wave of amplitude 3 has two complete cycles in the interval between $\theta = 0°$ and $\theta = 180°$. Write its equation.

37. $\boxed{y = A \sin \theta}$

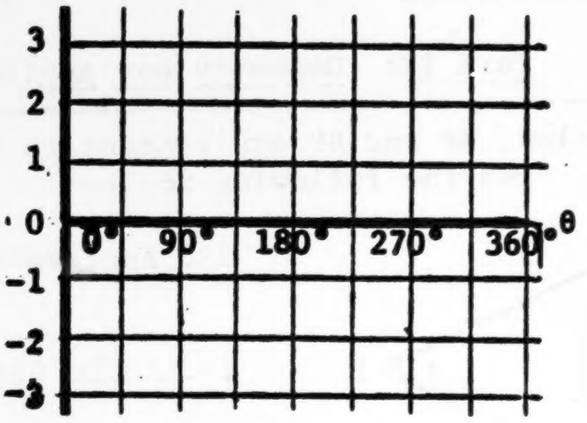
If $y = 20$ when $\theta = 150°$, find the numerical value of A.

36.

37. A =

38. On the axes at the right, sketch the graph of:

$$\boxed{y = 3 \sin 2\theta}$$



38. See graph.

Part VI: Quadratic and Radical Equations

The roots of the quadratic equation $\boxed{ax^2 + bx + c = 0}$ are:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

39-40. Find the numerical value of the two roots of:

$$\boxed{t^2 + 3t - 2 = 0}$$

39. t =

40. t =

41. Solve for w:

$$\boxed{3 + \sqrt{w - 2} = 5}$$

42. Solve for d:

$$\boxed{p = \sqrt{2bd}}$$

43. Solve for t:

$$\boxed{r = 2c\sqrt{\frac{a}{t}}}$$

41. w =

42. d =

43. t =

Part VII: Systems of Equations

44-45. Solve:

$$\begin{aligned} 5r + 3t &= 1 \\ 3r + 2t &= 2 \end{aligned}$$

44. $r =$

45. $t =$

46. Eliminate F and solve for B:

$$\begin{aligned} B &= FH \\ BF &= A \end{aligned}$$

47. Eliminate G and solve for R:

$$\begin{aligned} A &= G - R \\ R &= BG \end{aligned}$$

46. $B =$

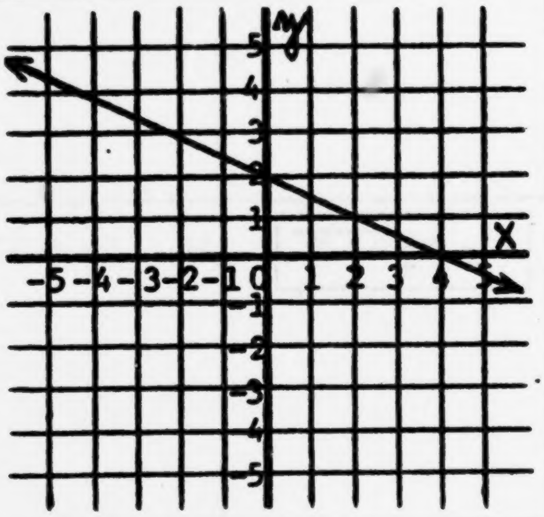
47. $R =$

Part VIII: Graphing and Slope

48. Refer to the coordinate system at the right. Determine the slope of the line whose graph is shown.

49. On the coordinate system at the right, graph this equation:

$$x - 2y = 4$$



48.

49. See graph.

50. Without graphing, determine the slope of this line:

$$3y - 4x = 12$$

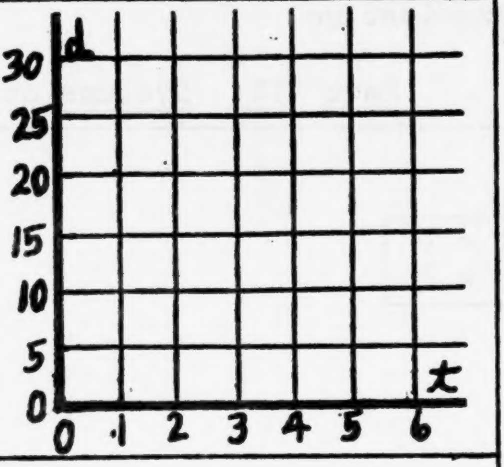
51. Find the slope of the straight line which passes through the points (1,-2) and (-1,4).

50. Slope =

51. Slope =

52. On the axes at the right, graph this formula:

$d + 4t = 20$



52. See graph.

53. Given this formula: $F = 20h$

If F changes by 10 units, find the corresponding change in h.

53. $\Delta h =$

Part IX: Logarithms and Exponentials

54. log 87,200 = ?

55. log 0.00307 = ?

54.

55.

56. Find N, if: $\log N = -1.9000$

57. $D = 20 \log R$

If D = 60, find R.

56. N =

57. R =

58. Find P, if: $P = \sqrt[5]{43.8}$

59. ln 4.53 = ?

58. P =

59.

60. Find M, if: $M = e^{10}$

61. Find t, if: $e^t = 200$

60. M =

61. t =

62. Write this equation in exponential form: $\boxed{\ln 5.90 = 1.7750}$

62.

63. If $t = 1.80$, $e^{-t} = \underline{\ ?\ }$

64. If $\ln N = 0.410$, $N = \underline{\ ?\ }$

63.

64. N =

65. Find D, if: $\boxed{D = 10 \log\left(\frac{R}{T}\right)}$

$R = 800$
$T = 400$

66. Find V, if: $\boxed{V = ke^{t}}$

$k = 100$
$t = 3.40$

65. D =

66. V =

67. Find G, if: $\boxed{G = A(1 - e^{-t})}$

$A = 200$
$t = 2.30$

68. Find W, if: $\boxed{W = \frac{H}{\ln H}}$

$H = 7.10$

67. G =

68. W =

IMPORTANT: The two remaining sections or parts are:

Part X(A): Complex Numbers
Part X(B): Oblique Triangles

Only one of these parts is to be worked.

If you are an Electrical Technology student, work Part X(A): Complex Numbers, and omit Part X(B): Oblique Triangles.

If you are not an Electrical student, work Part X(B): Oblique Triangles, and omit Part X(A): Complex Numbers.

Part X(A): Complex Numbers

Note: Work this part only if you are an Electrical Technology student.

69-A. Multiply. Write the product in complex number form.

$$(2 - j)(3 - 4j)$$

70-A. Divide. Write the quotient in complex number form.

$$\frac{3 - 2j}{1 - j}$$

69-A. ☐

70-A. ☐

71-A. Write this vector in complex number form.

$$10.0 \angle 310°$$

72-A. Write this vector in polar coordinate form.

$$3.00 - 4.00j$$

71-A. ☐

72-A. ☐

73-A. Divide. Write the quotient in polar coordinate form.

$$\frac{60.0 \angle 210°}{4.00 \angle -70°}$$

74-A. Multiply. Write the product in polar coordinate form.

$$(3.00 \angle 72°)(2.00 \angle 103°)$$

73-A. ☐

74-A. ☐

75-A. Given these vectors:

$$5.00 \angle 270° \text{ and } 4.00 \angle 0°$$

Find their resultant (sum) in complex number form.

76-A. Given: $Z_T = \dfrac{Z_1}{Z_1 + Z_2}$

Find Z_T in polar coordinate form, if:

$Z_1 = 20.0 \angle 0°$

$Z_2 = 30.0 \angle 180°$

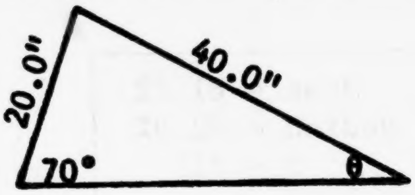
75-A. ☐

76-A. ☐

Part X(B): Oblique Triangles

Note: Work this part only if you are not an Electrical Technology student.

69-B. Set up the equation for calculating the size of angle θ.

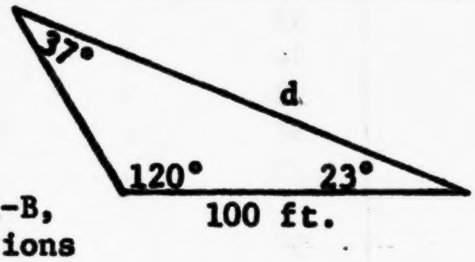


70-B. Using the equation set up in Problem 69-B, perform the calculations and find the size of angle θ in degrees.

69-B.

70-B. θ =

71-B. Set up the equation for calculating the length of side d.

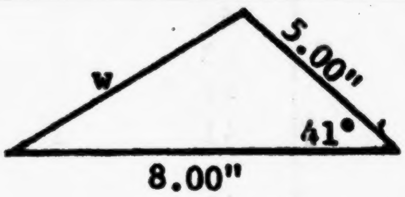


72-B. Using the equation set up in Problem 71-B, perform the calculations and find the length of side d.

71-B.

72-B. d =

73-B. Set up the equation for calculating the length of side w.

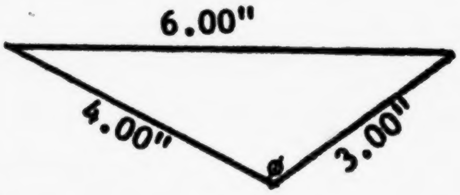


74-B. Using the equation set up in Problem 73-B, perform the calculations and find the length of side w.

73-B.

74-B. w =

75-B. Set up the equation for calculating the size of angle ϕ.



76-B. Using the equation set up in Problem 75-B, perform the calculations and find the size of angle ϕ in degrees.

75-B.

76-B. ϕ =

DISTRIBUTION OF SCORES FOR FINAL EXAMINATION
TECHNICAL MATHEMATICS 2 - MAY, 1969
MILWAUKEE AREA TECHNICAL COLLEGE

Mean = 81.2%
Median = 82.9%
N = 215

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent	Score	N	Percent of Students	Cumulative Percent
76							
75	3	1.3%	1.3%	55	5	2.3%	78.0%
74	2	0.9%	2.2%	54	7	3.3%	81.3%
73	7	3.3%	5.5%	53	5	2.3%	83.6%
72	7	3.3%	8.8%	52	4	1.9%	85.5%
71	12	5.6%	14.4%	51	4	1.9%	87.4%
70	8	3.7%	18.1%	50	1	0.5%	87.9%
69	13	6.0%	24.1%	49	4	1.9%	89.8%
68	18	8.4%	32.5%	48	1	0.5%	90.3%
67	12	5.6%	38.1%	47	2	0.9%	91.2%
66	8	3.7%	41.8%	46	3	1.3%	92.5%
65	8	3.7%	45.5%	45	5	2.3%	94.8%
64	6	2.7%	48.2%	44	2	0.9%	95.7%
63	14	6.5%	54.7%	43	4	1.9%	97.6%
62	7	3.3%	58.0%	42	2	0.9%	98.5%
61	10	4.7%	62.7%	41	1	0.5%	99.0%
60	7	3.3%	66.0%	40			
59	9	4.2%	70.2%	39			
58	6	2.7%	72.9%	38	1	0.5%	99.5%
57	4	1.9%	74.8%	37			
56	2	0.9%	75.7%	36	1	0.5%	100.0%

ITEM ANALYSIS FOR FINAL EXAMINATION TECHNICAL MATHEMATICS 2 - MAY, 1969 MILWAUKEE AREA TECHNICAL COLLEGE

Mean = 81.2%
Median = 82.9%
N = 215

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	%
Trig Ratios (Definition)	1.	95%
	2.	96%
	3.	92%
	4.	91%
	5.	95%
	6.	91%
Trig Ratios (Non-Acute Angles)	7.	89%
	8.	88%
	9.	88%
	10.	95%
Vectors	11.	37%
	12.	93%
	13.	84%
	14.	93%
	15.	79%
	16.	71%
Further Trig Topics	17.	88%
	18.	89%
	19.	96%
	20.	94%
	21.	88%
	22.	93%
	23.	94%
	24.	91%
	25.	80%
	26.	59%
	27.	60%
Geometry and Applied Trig	28.	95%
	29.	93%
	30.	91%
	31.	54%
	32.	85%
	33.	74%
Sine Waves	34.	97%
	35.	95%
	36.	80%
	37.	80%
	38.	94%
Quadratic and Radical Equations	39.	71%
	40.	72%
	41.	78%
	42.	96%
	43.	76%

Topic	Item No.	%
Systems of Equations	44.	91%
	45.	92%
	46.	94%
	47.	88%
Graphing and Slope	48.	65%
	49.	80%
	50.	79%
	51.	72%
	52.	94%
	53.	66%
Logarithms and Exponentials	54.	87%
	55.	70%
	56.	59%
	57.	47%
	58.	24%
	59.	68%
	60.	84%
	61.	95%
	62.	72%
	63.	92%
	64.	81%
	65.	81%
	66.	86%
	67.	84%
	68.	84%
Complex Numbers Electrical Students N = 84	69.	81%
	70.	48%
	71.	79%
	72.	71%
	73.	89%
	74.	95%
	75.	87%
	76.	50%
Oblique Triangles Non-Electrical Students N = 131	69.	95%
	70.	90%
	71.	86%
	72.	73%
	73.	81%
	74.	56%
	75.	73%
	76.	39%

APPENDIX I

DATA FOR COMPREHENSIVE ADVANCED ALGEBRA EXAM (MAY, 1969)

I-1 Copy of Comprehensive Exam: Advanced Algebra (Form B)
May, 1969

I-2 Distribution of Scores
Comprehensive Exam: Advanced Algebra - May, 1969
MATC Technical Mathematics (1968-69)

Item Analysis
Comprehensive Exam: Advanced Algebra - May, 1969
MATC Technical Mathematics (1968-69)

COMPREHENSIVE EXAM: ADVANCED ALGEBRA (Form B)

Part I: Equations Involving Radicals and Squares

1. Solve for W: $R^2 + W^2 = H^2$

2. Solve for s: $r = 2h\sqrt{\frac{a}{s}}$

3. Solve for d: $b = \frac{a(d^2 - t^2)}{pr}$

4. Solve for r: $\frac{7}{1 + 2\sqrt{r}} = 2$

1. $W =$

2. $s =$

3. $d =$

4. $r =$

Part II: Quadratic Equations

Note: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

5-6. Find the roots of:

$$4t^2 - 9 = 0$$

7-8. Find the roots of:

$$R^2 + 2R - 8 = 0$$

5. $t =$

6. $t =$

7. $R =$

8. $R =$

9-10. Find the roots of:

$$w + 1 = \frac{w}{w - 4}$$

9. w =

10. w =

Part III: Systems of Two Equations

11. Eliminate G and solve for P:

$$\begin{aligned} T &= G^2P \\ P &= \frac{N}{G} \end{aligned}$$

12. Eliminate T and solve for E:

$$\begin{aligned} W &= T - E \\ E &= RT \end{aligned}$$

11. P =

12. E =

13. Eliminate a and solve for h:

$$\begin{aligned} t &= \sqrt{h - a} \\ h &= \frac{a}{t} \end{aligned}$$

14-15. Solve for F and G:

$$\begin{aligned} F - 2G &= 5 \\ 2F - 3G &= 6 \end{aligned}$$

13. h =

14. F =

15. G =

16-17. Solve for t and w:

$$t = 10 - (t - w)$$
$$t + \frac{w}{2} = 9$$

16. $t =$

17. $w =$

Part IV: Systems of Three Equations

18-20. Solve for P_1, P_2, and P_3:

$$P_1 - P_2 + P_3 = 7$$
$$P_1 = 2P_2 + 7$$
$$P_3 = P_2 + 4$$

18. $P_1 =$

19. $P_2 =$

20. $P_3 =$

21. Eliminate h and r, and solve for d:

$$h = a - r$$
$$b + d = h$$
$$r = ad$$

21. $d =$

DISTRIBUTION OF SCORES
COMPREHENSIVE EXAM: ADVANCED ALGEBRA - MAY, 1969
MATC TECHNICAL MATHEMATICS (1968-69)

Mean = 71.4%
Median = 76.2%
N = 216

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
21	16	7.4%	7.4%
20	28	13.0%	20.4%
19	14	6.5%	26.9%
18	22	10.2%	37.1%
17	26	12.0%	49.1%
16	11	5.1%	54.2%
15	11	5.1%	59.3%
14	13	6.0%	65.3%
13	14	6.5%	71.8%
12	12	5.5%	77.3%
11	5	2.3%	79.6%
10	9	4.2%	83.8%
9	11	5.1%	88.9%
8	4	1.8%	90.7%
7	9	4.2%	94.9%
6	2	0.9%	95.8%
5	4	1.8%	97.6%
4	3	1.4%	99.0%
3			
2	1	0.5%	99.5%
1	1	0.5%	100.0%

ITEM ANALYSIS
COMPREHENSIVE EXAM: ADVANCED ALGEBRA - MAY, 1969
MATC TECHNICAL MATHEMATICS (1968-69)

Mean = 71.4%
Median = 76.2%
N = 216

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Topic-Unit Test	Comprehensive Exam
Equations Involving Radicals and Squares	1.	91%	81%
	2.	86%	62%
	3.	64%	68%
	4.	66%	51%
Quadratic Equations	5.	94%	75%
	6.	92%	74%
	7.	91%	76%
	8.	89%	74%
	9.	78%	49%
	10.	70%	42%
Systems of Two Equations and Formulas	11.	79%	70%
	12.	79%	84%
	13.	70%	73%
	14.	90%	87%
	15.	92%	90%
	16.	77%	79%
	17.	75%	76%
Systems of Three Equations and Formulas	18.	83%	81%
	19.	74%	74%
	20.	76%	73%
	21.	56%	60%

APPENDIX J

DATA FOR COMPREHENSIVE GRAPHING EXAM (MAY, 1969)

J-1 Copy of Comprehensive Exam: Graphing (Form B)
May, 1969

J-2 Distribution of Scores
Comprehensive Exam: Graphing - May, 1969
MATC Technical Mathematics (1968-69)

Item Analysis
Comprehensive Exam: Graphing - May, 1969
MATC Technical Mathematics (1968-69)

COMPREHENSIVE EXAM: GRAPHING (Form B)

Part I: Introduction to Graphing

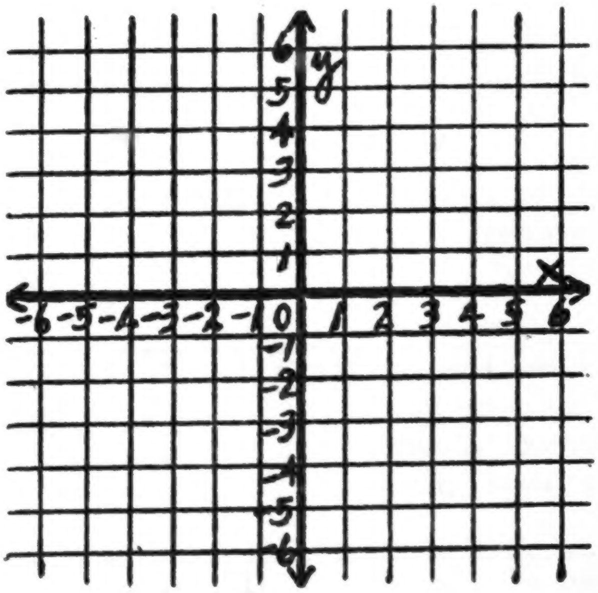
1-3. Complete the table of solutions for this equation:

$\boxed{y = x + 2}$

	x	y
1.	-6	
2.	-1	
3.	0	

4. On the axes at the right, construct the graph of:

$\boxed{y = x + 2}$


y
x
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

	x	y
1.	-6	
2.	-1	
3.	0	

4. See graph.

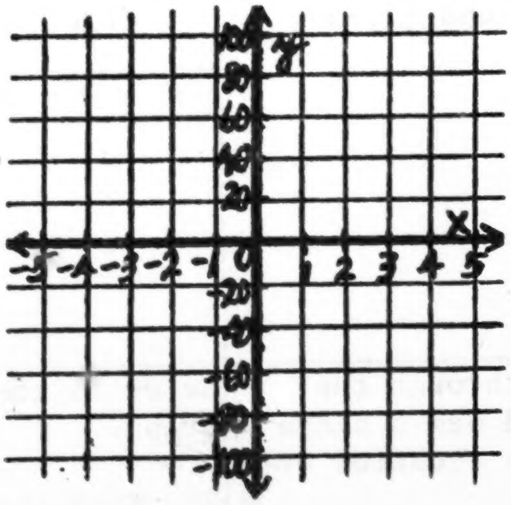
5-6. Complete the table of solutions for this equation:

$\boxed{y = 10x^2}$

	x	y
5.	-3	
6.	$1\frac{1}{2}$	

7. On the axes at the right, construct the graph of:

$\boxed{y = 10x^2}$


y
x
100
80
60
40
20
0
-20
-40
-60
-80
-100
-5 -4 -3 -2 -1 0 1 2 3 4 5

	x	y
5.	-3	
6.	$1\frac{1}{2}$	

7. See graph.

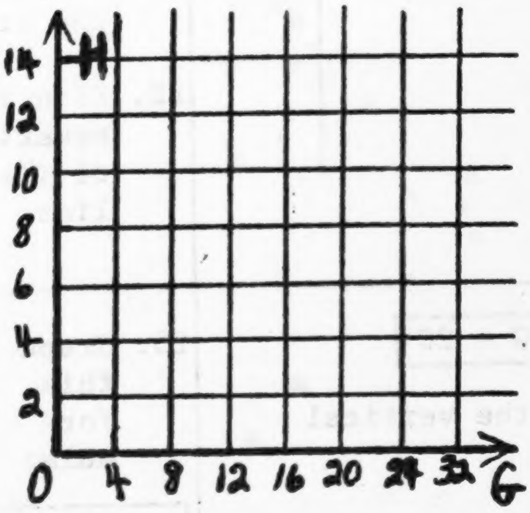
8. Given this formula: $\boxed{GH = 24}$

If G = 10, H = ___?___

9. On the axes at the right, construct the graph of:

$\boxed{GH = 24}$

(Plot G on the horizontal axis, and plot H on the vertical axis.)


H
14
12
10
8
6
4
2
0
4 8 12 16 20 24 32
G

8. H =

9. See graph.

10. $\boxed{3x - y = 8}$

If x = 1, y = ___?___

11. A point has a negative abscissa and a positive ordinate. In what quadrant does the point lie?

10. y =

11.

Part II: Straight Line and Slope

Given: $r + 3s = 6$

If r is plotted on the vertical axis:

12. Find the coordinates of the horizontal intercept.

13. Find the coordinates of the vertical intercept.

Given: $P + 4W = 8$

If P is plotted on the vertical axis:

14. Find the coordinates of the vertical intercept.

15. Find the slope of the line.

12.

13.

14.

15. Slope =

16. State the definition of slope using Δh and Δv, where Δh means "horizontal change" and Δv means "vertical change."

17. The slope of a line is 6. For a vertical change of +2 units, what is the corresponding horizontal change?

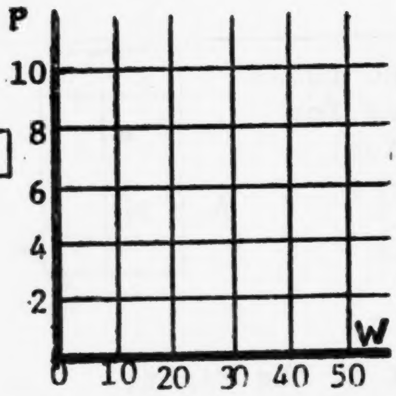
16. Slope =

17. Δh =

18. Find the slope of the straight line passing through (-1,3) and (1,-3).

19. Graph this formula:

$3P + W = 30$



18. Slope =

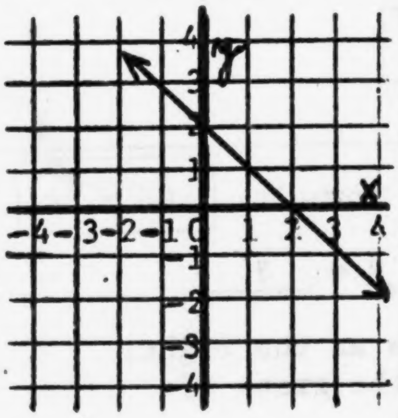
19. See graph.

20. A line passes through the point (2,3) and has a slope of 3. Find the equation of the line.

Refer to the graph.

21. Find the slope of the line.

22. Find the equation of the line.



20.

21. Slope =

22.

Given: $F - 5G = 20$

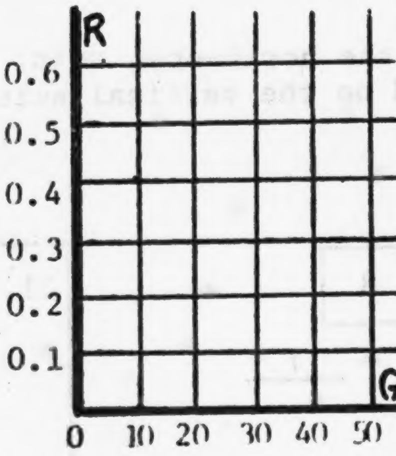
F is plotted on the vertical axis.

23. Write the equation in slope-intercept form.

24. If G increases by 2 units, find the corresponding change in F.

25. Graph this formula:

$R = 0.025G$



23.

24. ΔF =

25. See graph.

Part III: Sine Wave Graphs

Directions: Where necessary, refer to a "Trig Ratios" table.

26. $\boxed{y = -50 \sin \theta}$

If $\theta = -300°$,
$y = ?$

27. $\boxed{y = 100 \sin(\theta - 120°)}$

If $\theta = -90°$,
$y = ?$

26. y =

27. y =

28. A sine wave of amplitude 250 lags the fundamental sine wave by 42°. Write its equation.

29. Write the equation of the third harmonic whose amplitude is 0.35.

28.

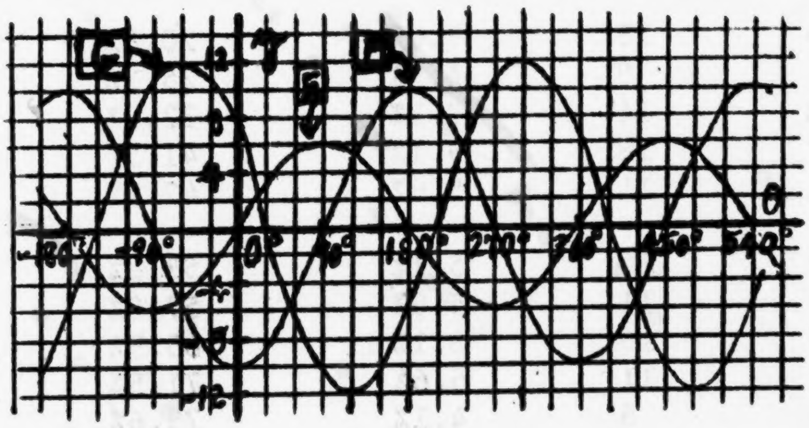
29.

Refer to the diagram.
Write the equation of:

30. Sine wave E.

31. Sine wave F.

32. Sine wave G.



30.

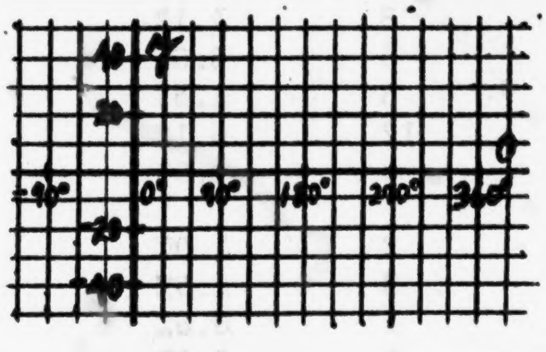
31.

32.

On the axes at the right, carefully sketch the graph of:

33. $y = 40 \sin \theta$

34. $y = 20 \sin 2\theta$



33. See sketch.

34. See sketch.

Part IV: Exponential Graphs

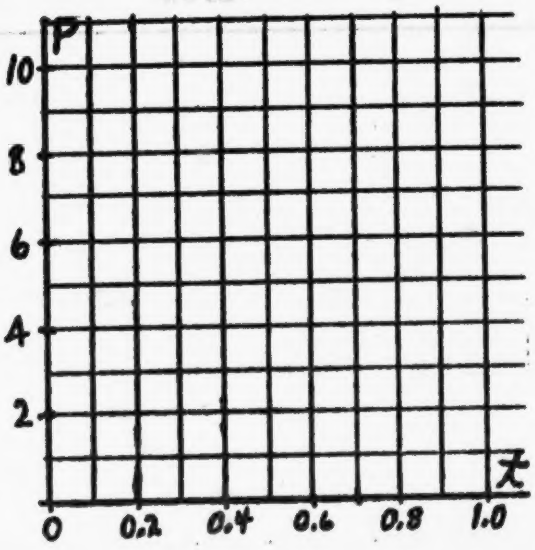
Directions: Where necessary, refer to the "Table of e^x and e^{-x}."

Given: $\boxed{P = 10e^{-2t}}$

35. If t = 0, P = ___?___

36. If t = 0.1, P = ___?___

37. Graph the equation.



35. P =

36. P =

37. See graph.

DISTRIBUTION OF SCORES
COMPREHENSIVE EXAM: GRAPHING - MAY, 1969
MATC TECHNICAL MATHEMATICS (1968-69)

Mean = 78.9%
Median = 81.1%
N = 197

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
37	10	5.1%	5.1%
36	16	8.1%	13.2%
35	15	7.6%	20.8%
34	9	4.5%	25.3%
33	14	7.1%	32.4%
32	18	9.1%	41.5%
31	11	5.6%	47.1%
30	10	5.1%	52.2%
29	11	5.6%	57.8%
28	16	8.1%	65.9%
27	11	5.6%	71.5%
26	8	4.1%	75.6%
25	8	4.1%	79.7%
24	6	3.0%	82.7%
23	3	1.5%	84.2%
22	12	6.1%	90.3%
21	6	3.0%	93.3%
20	2	1.0%	94.3%
19	3	1.5%	95.8%
18	1	0.6%	96.4%
17	1	0.6%	97.0%
16	2	1.0%	98.0%
15	2	1.0%	99.0%
14	2	1.0%	100.0%

APPENDIX J-2

ITEM ANALYSIS
COMPREHENSIVE EXAM: GRAPHING - MAY, 1969
MATC TECHNICAL MATHEMATICS (1968-69)

Mean = 78.9%
Median = 81.1%
N = 197

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Topic-Unit Test	Comprehensive Exam
Graphing Simple Equations and Formulas	1.	100%	98%
	2.	100%	99%
	3.	100%	99%
	4.	98%	98%
	5.	98%	95%
	6.	90%	92%
	7.	89%	59%
	8.	99%	98%
	9.	92%	83%
	10.	90%	91%
	11.	99%	96%
Straight Line: Intercepts and Slope	12.	88%	66%
	13.	93%	68%
	14.	93%	77%
	15.	90%	65%
	16.	98%	84%
	17.	72%	64%
	18.	83%	57%
	19.	95%	90%
	20.	66%	49%
	21.	83%	68%
	22.	66%	55%
	23.	94%	82%
	24.	73%	63%
	25.	78%	89%
Sine Wave Graphs	26.	76%	61%
	27.	73%	73%
	28.	88%	68%
	29.	95%	87%
	30.	96%	90%
	31.	81%	68%
	32.	52%	48%
	33.	96%	92%
	34.	89%	87%
Exponential Graphs	35.	97%	89%
	36.	97%	92%
	37.	96%	79%

APPENDIX K

DATA FOR COMPREHENSIVE TRIGONOMETRY EXAM (MAY, 1969)

K-1 Copy of Comprehensive Exam: Trigonometry (Form A)
May, 1969

K-2 Distribution of Scores
Comprehensive Exam: Trigonometry - May, 1969
MATC Technical Mathematics (1968-69)

Item Analysis
Comprehensive Exam: Trigonometry - May, 1969
MATC Technical Mathematics (1968-69)

Milwaukee Area Technical College
Technical Mathematics Project

APPENDIX K-1

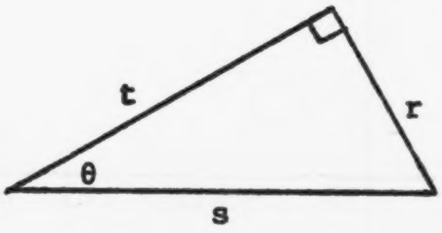
COMPREHENSIVE EXAM: TRIGONOMETRY (Form A)

Directions: A table of "Trig Ratios" will be provided. Calculations should be done on your slide rule.

Part I: Definitions and Solution of Right Triangles

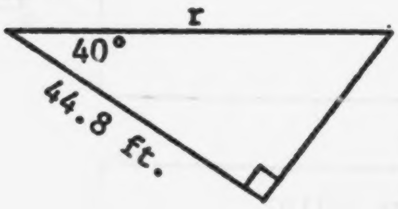
Using the right triangle shown, define the following trig ratios of angle θ:

1. cos θ = ?
2. tan θ = ?
3. sin θ = ?
4. sec θ = ?
5. cot θ = ?
6. csc θ = ?

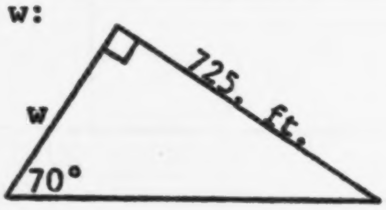


1. cos θ =
2. tan θ =
3. sin θ =
4. sec θ =
5. cot θ =
6. csc θ =

7. Find side r:



8. Find side w:



7. r = ft.

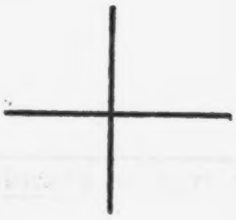
8. w = ft.

Part II: Trig Ratios of General Angles

In Problems 9 to 11, find the numerical value of each.

9. tan 98° = ?

10. sin 220° = ?



11. cos 330° = ?

9.

10.

11.

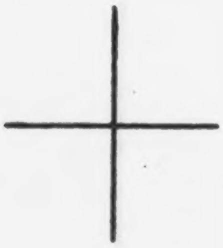
12. Which of the following angles have the same terminal side?

(a) 600° (b) -240° (c) 240° (d) -120°

12.

13. Find the numerical value of:

sin(-35°) = ?



14. Find the numerical value of:

csc 208° = ?

15-16. If sin θ = -0.866, find two values of angle θ lying between 0° and 360°.

13.

14.

15. θ =

16. θ =

Part III: Further Trig Topics

17. Write this equation using arctan notation:

$\tan K = G$

18. Write this equation in regular trigonometric form:

$R = \sin^{-1}P$

17.

18.

19. Convert to degrees:

3.49 radians

20. Convert to radians:

13°

19.

20.

21. Complete this identity:

$\sin^2\theta + \underline{\ \ ?\ \ } = 1$

22. Complete this identity:

$\tan\theta = \dfrac{?}{\cos\theta}$

21.

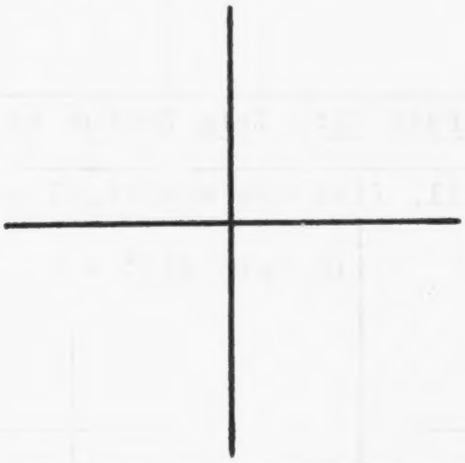
22.

Part IV: Vectors

A slanted vector begins at the origin and ends at the point (10.0, -30.0).

23. Find the length of the vector.

24. Find the standard position angle of the vector.



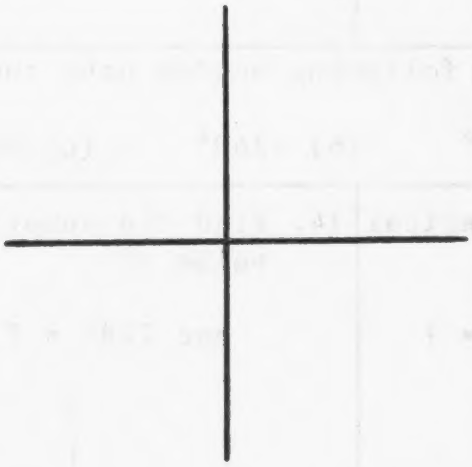
23.

24.

A slanted vector is 4.00 units long and has a standard position angle of 236°. Find the following:

25. The horizontal component of the vector.

26. The vertical component of the vector.



25.

26.

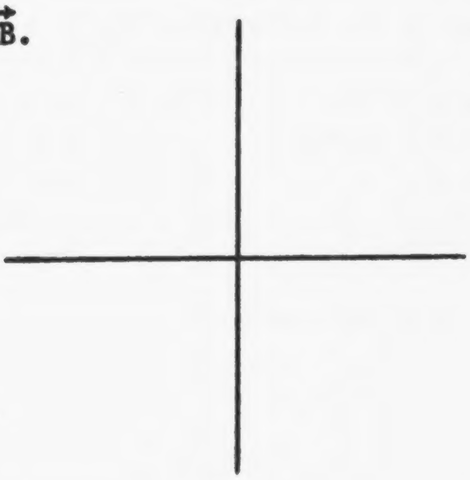
A vector system has two vectors, $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Vector $\overrightarrow{OA}$ has these components:

Horizontal: -32.7 units
Vertical: 46.9 units

Vector $\overrightarrow{OB}$ has these components:

Horizontal: 65.4 units
Vertical: -21.6 units



These two vectors have a resultant.

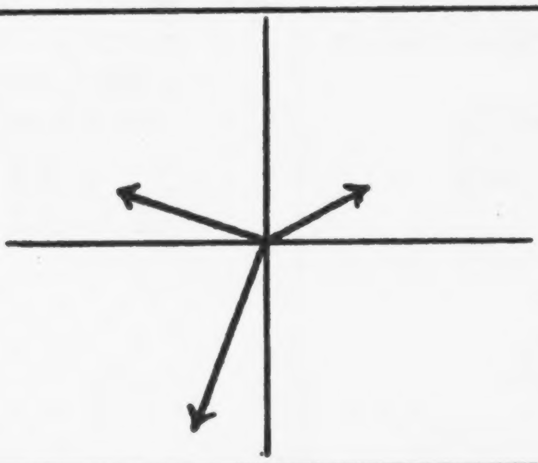
27. Find the horizontal component of the resultant.

28. Find the vertical component of the resultant.

27.

28.

29. On the diagram at the right, sketch the resultant of the three vectors shown.

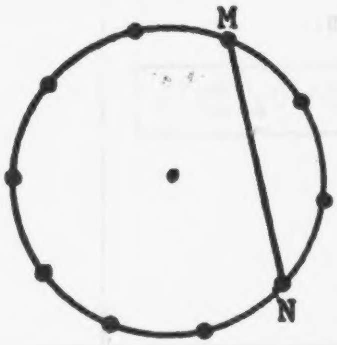


29. See sketch.

Part V: Applied Problem

30. Ten equally-spaced holes are placed on the circumference of a circle whose diameter is 20.0". Refer to the diagram at the right.

Find the length of chord MN.



30.

Note: Parts VI and VII follow on the next two pages.

If you are an Electrical Technology student, work only Part VI, Complex Numbers. Omit Part VII.

If you are not an Electrical Technology student, work only Part VII, Oblique Triangles. Omit Part VI.

Part VI: Complex Numbers

Note: This section is to be worked only by Electrical Technology students.

31-E. Write this vector in polar coordinate form:

$\boxed{5.00 - 8.00j}$

32-E. Write this vector in complex number form:

$\boxed{10.0\angle 130^\circ}$

31-E.

32-E.

33-E. Multiply, and write the product in complex number form:

$\boxed{(1 - j)(4 - 2j)}$

34-E. Divide, and write the quotient in complex number form:

$\boxed{\frac{1 - 3j}{2 - j}}$

33-E.

34-E.

35-E. Multiply, and write the product in polar coordinate form:

$\boxed{(20.0\angle 40^\circ)(30.0\angle 70^\circ)}$

36-E. Divide, and write the quotient in polar coordinate form:

$\boxed{\frac{695\angle 120^\circ}{292\angle -40^\circ}}$

35-E.

36-E.

Given these two vectors: $\boxed{200\angle 180^\circ \quad \text{and} \quad 150\angle 90^\circ}$

37-E. Find their resultant in complex number form.

38-E. Find their resultant in polar coordinate form.

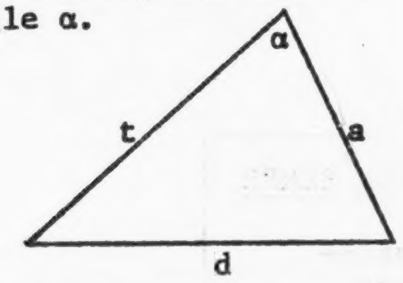
37-E.

38-E.

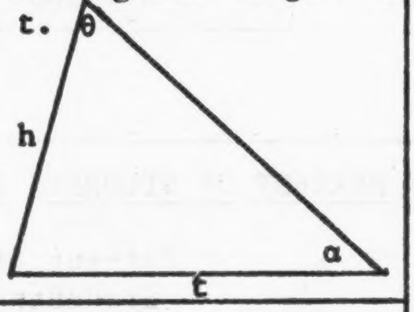
Part VII: Oblique Triangles

Note: This section is not to be worked by Electrical Technology students.

31-N. In the oblique triangle shown, side a, side d, and side t, are known. Set up the equation for calculating the size of angle α.



32-N. In the oblique triangle shown, angle α, angle θ, and side h are known. Set up the equation for calculating the length of side t.

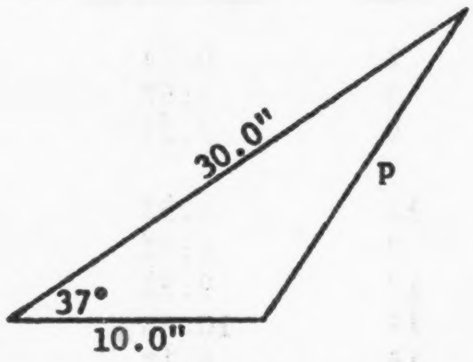


31-N.

32-N.

33-N. Set up the equation for calculating the length of side p.

34-N. Using the above equation, find the numerical length of side p.

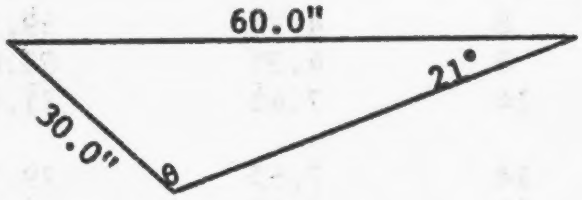


33-N.

34-N. p =

35-N. Set up the equation for finding angle θ.

36-N. Using the above equation, find the size of angle θ in degrees. Note that θ is an obtuse angle.

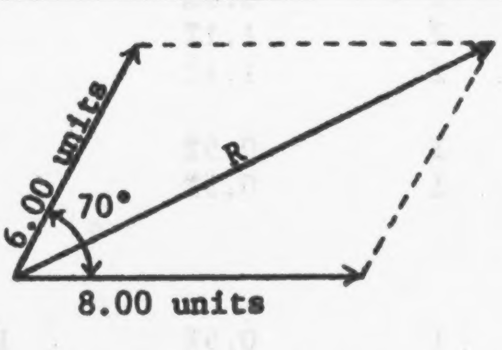


35-N.

36-N. θ =

37-N. Two vectors and their resultant are shown at the right. Set up the equation for finding resultant R.

38-N. Using the above equation, find the length of resultant R.



37-N.

38-N. R =

APPENDIX K-2

DISTRIBUTION OF SCORES
COMPREHENSIVE EXAM: TRIGONOMETRY - MAY, 1969
MATC TECHNICAL MATHEMATICS (1968-69)

Mean = 74.8%
Median = 76.3%
N = 185

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
38	1	0.5%	0.5%
37	3	1.6%	2.1%
36	4	2.2%	4.3%
35	12	6.5%	10.8%
34	9	4.9%	15.7%
33	17	9.2%	24.9%
32	19	10.3%	35.2%
31	15	8.1%	43.3%
30	8	4.3%	47.6%
29	14	7.6%	55.2%
28	8	4.3%	59.5%
27	8	4.3%	63.8%
26	14	7.6%	71.4%
25	14	7.6%	79.0%
24	10	5.4%	84.4%
23	6	3.2%	87.6%
22	4	2.2%	89.8%
21	2	1.1%	90.9%
20	4	2.2%	93.1%
19	5	2.7%	95.8%
18	1	0.5%	96.3%
17	2	1.1%	97.4%
16	2	1.1%	98.5%
15	1	0.5%	99.0%
14	1	0.5%	99.5%
13			
12			
11			
10	1	0.5%	100.0%

ITEM ANALYSIS
COMPREHENSIVE EXAM: TRIGONOMETRY - MAY, 1969
MATC TECHNICAL MATHEMATICS (1968-69)

Mean = 74.8%
Median = 76.3%
N = 185

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Topic-Unit Test	Comprehensive Exam
Right Triangles	1.	99%	97%
	2.	99%	97%
	3.	98%	97%
	4.	98%	91%
	5.	96%	96%
	6.	98%	91%
	7.	85%	85%
	8.	80%	89%
General Angles	9.	95%	84%
	10.	95%	86%
	11.	96%	89%
	12.	85%	66%
	13.	92%	87%
	14.	95%	49%
	15.	80%	71%
	16.	82%	76%
Arcsin Notation, Radians, Identities	17.	84%	82%
	18.	82%	78%
	19.	80%	86%
	20.	88%	80%
	21.	89%	83%
	22.	98%	92%
Vectors	23.	90%	82%
	24.	85%	52%
	25.	59%	37%
	26.	62%	41%
	27.	95%	80%
	28.	98%	85%
	29.	76%	70%
Applied Problem	30.	65%	66%
Complex Numbers (Electrical Students)	31-E.	69%	71%
	32-E.	85%	65%
	33-E.	97%	60%
	34-E.	82%	31%
	35-E.	82%	94%
	36-E.	92%	79%
	37-E.	92%	62%
	38-E.	65%	43%
Oblique Triangles (Non-Electrical Students)	31-N.	97%	61%
	32-N.	98%	83%
	33-N.	98%	70%
	34-N.	85%	94%
	35-N.	99%	81%
	36-N.	76%	30%
	37-N.	83%	44%
	38-N.	57%	28%

APPENDIX L

DATA FOR 20-ITEM PRE-TEST IN ALGEBRA (1967-68)

L-1 Copy of 20-Item Pre-Test in Algebra

L-2 Distribution of Scores for 20-Item Pre-Test in Algebra
Pius XI High School - Technical Mathematics (January, 1968)
(Juniors and Seniors)

Item Analysis for 20-Item Pre-Test in Algebra
Pius XI High School - Technical Mathematics (January, 1968)
(Juniors and Seniors)

L-3 Distribution of Scores for 20-Item Pre-Test in Algebra
Administered Three Different Times (January, March, & June, 1968)
Pius XI High School - Technical Mathematics
(Juniors and Seniors)

Item Analysis for 20-Item Pre-Test in Algebra
Administered Three Different Times (January, March, & June, 1968)
Pius XI High School - Technical Mathematics
(Juniors and Seniors)

L-4 Distribution of Scores for 20-Item Pre-Test in Algebra
Conventional Algebra Classes - Pius XI High School (May, 1968)
Experimental Tech Math Class - Pius XI High School (Jan., March, June, 1968)
MATC Tech Math Classes (Sept., Oct., 1967)

Item Analysis for 20-Item Pre-Test in Algebra
Conventional Algebra Classes - Pius XI High School (May, 1968)
Experimental Tech Math Class - Pius XI High School (Jan., March, June, 1968)
MATC Tech Math Classes (Sept., Oct., 1967)

PRE-TEST IN ALGEBRA

Directions: Work each problem in the space provided. Show all necessary work. Do not use any separate scratch paper. Write your answers in the boxes. The time for the test is one period (50 minutes).

1. Simplify:

$(-5) - 9 = ?$

2. Simplify:

$3 + (-2) - 7 - (-9) = ?$

3. Simplify:

$(4)(-5)(0)(2) = ?$

4. Complete:

$$\frac{\frac{5}{8}}{\frac{3}{4}} = ?$$

5. Solve for x:

$8x - (2 + x) = 19$

6. Solve for y:

$42 = 7 - 5(y + 1)$

7. Solve for R:

$5 = R - 2(1 - 3R)$

8. Solve for h:

$20 - (4 + 3h) = 10 - 5(2 - h)$

1. ☐
2. ☐
3. ☐
4. ☐
5. x =
6. y =
7. R =
8. h =

9. Solve for x:

$$6 = \frac{7}{4x}$$

10. Solve for w:

$$\frac{6w - 11}{w + 3} = 0$$

11. Solve for t:

$$\frac{2}{3t} = \frac{t - 1}{t}$$

12. Solve for y:

$$\frac{y}{2} - 5 = \frac{2y}{3}$$

13. Solve for x:

$$\frac{7}{x} = \frac{9}{2x} - \frac{5}{6}$$

14. Solve for P:

$$t = \frac{W}{P}$$

9. x =

10. w =

11. t =

12. y =

13. x =

14. P =

15. Solve for V_1:

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

16. Solve for G:

$$M = K - G$$

15. V_1 =

16. G =

17. Solve for a:

$$V_1 = V_2 - at$$

18. Solve for M:

$$G = \frac{L - M}{P}$$

17. a =

18. M =

19. Solve for A:

$$B = \frac{A}{1 - A}$$

20. Solve for H:

$$\frac{1}{F} = \frac{1}{G} - \frac{1}{H}$$

19. A =

20. H =

APPENDIX L-2

DISTRIBUTION OF SCORES FOR 20-ITEM PRE-TEST IN ALGEBRA PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (JANUARY, 1968) (JUNIORS AND SENIORS)

Mean = 36.2%
Median = 35.0%
N = 31

NUMBER AND PERCENT OF STUDENTS ACHIEVING EACH SCORE

Score	N	Percent of Students	Cumulative Percent
20			
19			
18			
17			
16			
15			
14			
13	1	3.2%	3.2%
12	2	6.5%	9.7%
11	4	12.8%	22.5%
10	2	6.5%	29.0%
9	4	12.8%	41.8%
8	2	6.5%	48.3%
7	2	6.5%	54.8%
6	2	6.5%	61.3%
5	5	16.1%	77.4%
4	3	9.7%	87.1%
3	2	6.5%	93.6%
2	1	3.2%	96.8%
1			
0	1	3.2%	100.0%

ITEM ANALYSIS FOR 20-ITEM PRE-TEST IN ALGEBRA PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (JANUARY, 1968) (JUNIORS AND SENIORS)

Mean = 36.2%
Median = 35.0%
N = 31

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	%
Signed Numbers	1.	71%
	2.	61%
	3.	74%
Division of Fractions	4.	71%
Non-Fractional Equations	5.	39%
	6.	32%
	7.	55%
	8.	16%
Fractional Equations	9.	36%
	10.	16%
	11.	29%
	12.	32%
	13.	19%
Formula Rearrangement	14.	42%
	15.	26%
	16.	64%
	17.	16%
	18.	26%
	19.	0%
	20.	0%

APPENDIX L-3

DISTRIBUTION OF SCORES FOR 20-ITEM PRE-TEST IN ALGEBRA ADMINISTERED THREE DIFFERENT TIMES (JANUARY, MARCH, & JUNE, 1968)
PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS
(JUNIORS AND SENIORS)

(Note: The same students, totaling 31, took all three tests.)

	Pre-Test Jan. 1968	Retest Mar. 1968	Retest June 1968
Mean	36.2%	88.8%	82.0%
Median	35.0%	90.0%	85.0%
N	31	31	31

NUMBER AND CUMULATIVE PERCENT OF STUDENTS ACHIEVING EACH SCORE

	Pre-Test Jan. 1968		Retest Mar. 1968		Retest June 1968	
Score	N	Cum. %	N	Cum. %	N	Cum. %
20			2	6.5%	2	6.5%
19			12	45.1%	5	22.7%
18			7	67.7%	6	42.0%
17			2	74.2%	3	51.7%
16			5	90.3%	6	71.0%
15			1	93.5%	3	80.7%
14			2	100.0%	3	90.4%
13	1	3.2%				
12	2	9.7%			1	93.6%
11	4	22.5%			1	96.8%
10	2	29.0%				
9	4	41.8%				
8	2	48.3%			1	100.0%
7	2	54.8%				
6	2	61.3%				
5	5	77.4%				
4	3	87.1%				
3	2	93.6%				
2	1	96.8%				
1						
0	1	100.0%				

ITEM ANALYSIS FOR 20-ITEM PRE-TEST IN ALGEBRA ADMINISTERED THREE DIFFERENT TIMES (JANUARY, MARCH, & JUNE, 1968) PIUS XI HIGH SCHOOL - TECHNICAL MATHEMATICS (JUNIORS AND SENIORS)

(Note: The same students, totaling 31, took all three tests.)

	Pre-Test Jan. 1968	Retest Mar. 1968	Retest June 1968
Mean	36.2%	88.8%	82.0%
Median	35.0%	90.0%	85.0%
N	31	31	31

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

Topic	Item No.	Pre-Test Jan. 1968	Retest Mar. 1968	Retest June 1968	Gain From Jan. 1968 to June 1968
Signed Numbers	1.	71%	97%	87%	+16%
	2.	61%	94%	83%	+22%
	3.	74%	100%	100%	+26%
Division of Fractions	4.	71%	87%	83%	+12%
Non-Fractional Equations	5.	39%	84%	80%	+41%
	6.	32%	81%	87%	+55%
	7.	55%	84%	97%	+42%
	8.	16%	78%	70%	+54%
Fractional Equations	9.	36%	94%	93%	+57%
	10.	16%	72%	60%	+44%
	11.	29%	81%	43%	+14%
	12.	32%	94%	77%	+45%
	13.	19%	77%	73%	+54%
Formula Rearrangement	14.	42%	97%	97%	+55%
	15.	26%	100%	97%	+71%
	16.	64%	94%	87%	+23%
	17.	16%	97%	90%	+74%
	18.	26%	87%	87%	+61%
	19.	0%	84%	80%	+80%
	20.	0%	97%	70%	+70%

DISTRIBUTION OF SCORES FOR 20-ITEM PRE-TEST IN ALGEBRA:
CONVENTIONAL ALGEBRA CLASSES - PIUS XI HIGH SCHOOL (MAY, 1968)
EXPERIMENTAL TECH MATH CLASS - PIUS XI HIGH SCHOOL (JAN., MARCH, JUNE, 1968)
MATC TECH MATH CLASSES (SEPT., OCT., 1967)

(Note: Means and medians, expressed as percents, are at the bottom of the table.)

	Conventional High School Algebra Classes													Pius XI Experimental Tech Math Class			MATC Tech Math Classes	
	Pius XI Freshmen							Pius XI Juniors										
	Ability Levels							Ability Levels						Pre-Test	Re-Test	Final Test	Pre-Test	Re-Test
Score	1	1	2	3	4	5	Total	1	1	2	2	3	Total					
20								2	1				3		2	2	5	178
19								1	3				4		12	5	6	108
18		2					2	6	4		1		11		7	6	9	61
17	4	1	1				6	4	7	1			12		2	3	14	25
16	3	1					4	2	4	1	2		9		5	6	20	16
15	5	2	1				8	4		2	2		8		1	3	14	7
14	1	3		1			5	5	2				7		2	3	23	2
13	3	4	3	1			11	2	4	3	1	1	11	1			13	1
12	3	2	4				9	2		2	5	1	10	2		1	11	1
11	4	2	3	1			10		2	3		1	6	4		1	22	
10	2	3		1			6	1	3	5	6	1	16	2			11	
9	2	5	2				9			3	3	1	7	4			15	1
8	2	2	3	5			12	1		2	1	3	7	2		1	32	1
7	3	2		2	1		8	1			5	2	8	2			26	
6		2	4	1	4		11			2	1		3	2			26	
5		1	7	2	5		15					3	3	5			34	
4		1	2	5	4		12					3	3	3			25	1
3			1	8	10	2	21			1		3	4	2			31	
2			1	2	3	6	12					1	1	1			38	
1			2	4	5	9	20				2	2	4				19	
0				1	4	14	19							1			8	
N	32	33	34	34	36	31	200	31	30	25	29	22	137	31	31	31	402	402
Mean	62%	56%	40%	25%	16%	4%	33.5%	76%	78%	54%	50%	30%	59.5%	36%	89%	82%	41%	94%
Median	65%	55%	40%	20%	15%	5%	30.0%	75%	85%	50%	50%	25%	60.0%	35%	90%	85%	35%	95%

ITEM ANALYSIS FOR 20-ITEM PRE-TEST IN ALGEBRA:
CONVENTIONAL ALGEBRA CLASSES - PIUS XI HIGH SCHOOL (MAY, 1968)
EXPERIMENTAL TECH MATH CLASS - PIUS XI HIGH SCHOOL (JAN., MARCH, JUNE, 1968)
MATC TECH MATH CLASSES (SEPT., OCT., 1967)

PERCENT OF STUDENTS WORKING EACH ITEM CORRECTLY

	Conventional High School Algebra Classes													Pius XI Experimental Tech Math Class			MATC Tech Math Classes	
	Pius XI Freshmen							Pius XI Juniors										
Item No.	Ability Levels						Over-all %	Ability Levels					Over-all %	Pre-Test	Re-Test	Final Test	Pre-Test	Re-Test
	1	1	2	3	4	5		1	1	2	2	3						
1	100%	70%	83%	74%	58%	26%	69%	97%	97%	100%	86%	73%	91%	71%	97%	87%	58%	96%
2	97%	76%	71%	47%	39%	6%	56%	100%	100%	84%	72%	59%	85%	61%	94%	83%	55%	96%
3	97%	91%	88%	82%	17%	36%	68%	97%	100%	100%	86%	82%	93%	74%	100%	100%	53%	98%
4	91%	73%	53%	29%	39%	3%	48%	90%	97%	96%	90%	64%	88%	71%	87%	83%	58%	92%
5	84%	79%	56%	41%	50%	10%	54%	94%	100%	92%	59%	68%	83%	39%	84%	80%	70%	97%
6	69%	70%	38%	9%	28%	0%	36%	77%	73%	48%	55%	18%	53%	32%	81%	87%	46%	92%
7	78%	79%	53%	38%	42%	3%	49%	90%	90%	68%	76%	54%	77%	55%	84%	97%	58%	94%
8	66%	64%	53%	9%	28%	0%	37%	74%	77%	68%	59%	41%	65%	16%	78%	70%	46%	93%
9	75%	70%	47%	12%	3%	3%	35%	90%	87%	60%	45%	32%	65%	36%	94%	93%	42%	99%
10	53%	82%	29%	24%	0%	0%	31%	71%	63%	36%	28%	9%	44%	16%	72%	60%	18%	80%
11	38%	36%	35%	29%	0%	0%	23%	74%	70%	36%	21%	14%	45%	29%	81%	43%	22%	95%
12	47%	67%	47%	21%	6%	0%	31%	64%	70%	24%	45%	9%	45%	32%	94%	77%	27%	95%
13	25%	33%	41%	21%	0%	0%	20%	68%	67%	12%	31%	4%	39%	19%	77%	73%	21%	93%
14	78%	52%	18%	12%	6%	0%	27%	97%	97%	56%	69%	19%	71%	42%	97%	97%	56%	98%
15	50%	15%	18%	0%	0%	0%	14%	90%	90%	40%	34%	4%	55%	26%	100%	97%	39%	99%
16	78%	76%	29%	32%	0%	0%	35%	87%	93%	72%	69%	36%	74%	64%	94%	87%	59%	98%
17	50%	27%	9%	9%	0%	0%	16%	52%	67%	28%	48%	9%	43%	16%	97%	90%	35%	91%
18	66%	45%	9%	3%	0%	0%	20%	58%	70%	40%	28%	9%	43%	26%	87%	87%	36%	94%
19	0%	0%	9%	0%	0%	0%	2%	26%	23%	4%	3%	0%	12%	0%	84%	80%	8%	92%
20	6%	3%	3%	0%	0%	0%	2%	26%	17%	0%	3%	0%	10%	0%	97%	70%	10%	83%